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TORONTO

AN INTRODUCTORY LOGIC

BY

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IN CORNELL UNIVERSITY

Fifth Edition, Thoroughly Revised with the Addition of New Problems and Examples

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PREFACE TO THE FIFTH EDITION

In undertaking another revision of this famous text-book I have naturally endeavored to preserve the spirit of the original work, while modifying it considerably in the letter. The criticisms and suggestions of users of the book have been most helpful in this respect.

The exercises are practically new. They follow the promptings of the text less closely than did the old ones, and they are of varying degrees of difficulty. Thus it is hoped that in working them the student will have somewhat more opportunity to display his grasp of principles than was formerly the case. I have omitted certain portions of the older text because they were concerned with issues now more or less out of date, and I have substituted in their stead new sections dealing with recent developments. Thus for example parts of sections 2 and 90 of the fourth edition are left out of this edition because they had to do with psychological problems now stated in other terms. And the two chapters (XVI and XVII) on The Determination of Causal Relations are here condensed into one, in order thereby to emphasize by comparison the significance of the other aids to inductive inference, such as Analogy, etc. On the other hand the historical Chapter II has been slightly expanded, and sections 31, on Immediate Inference by Converse Relation, and 47, on Systematic Deduction, have been added. Sections 12, 18 and 68 are also new, while all

of the final chapter of the earlier edition, except section 99, now appended to Chapter XXIV, has been omitted. Other lesser omissions and additions occur too frequently for separate mention.

Besides these major changes numerous verbal and typographical alterations have been made, with the hope that they will add to the attractiveness and usefulness of the book without detracting anything from the simplicity and clarity which were among the outstanding characteristics of the earlier editions. In this connection my colleague, E. T. Paine, has rendered invaluable assistance.

HAROLD R. SMART.

CORNELL UNIVERSITY,
April, 1932.

PREFACE TO THE FIRST EDITION

This volume is intended primarily as a text-book for college students, and grew out of my lectures on logic to undergraduate classes in Cornell University. It aims at being both practical and theoretical. In spite of the obvious deficiencies of formal logic as a theory of the nature of thought, I am convinced that it is one of the most valuable instruments in modern education for promoting clear thinking, and for developing critical habits of mind. J. S. Mill, speaking in the *Autobiography* of the discipline which he received from working logical exercises, expresses the following opinion: "I am persuaded that nothing, in modern education, tends so much, when properly used, to form exact thinkers, who attach a precise meaning to words and propositions, and are not imposed on by vague, loose, or ambiguous terms." Although in treating the syllogistic logic I have followed to a large extent the ordinary mode of presentation, I have both here, and when dealing with the inductive methods, endeavored to interpret the traditional doctrines in a philosophical way, and to prepare for the theoretical discussions of the third part of the book.

The advisability of attempting to include a theory of thought, or philosophy of knowledge, even in outline, in an elementary course in logic, may at first sight appear doubtful. It seems to me, however, that this inclusion is not only justifiable, but even necessary at the present time.

Psychology is no longer a 'philosophy of mind'; but, under the influence of experimental methods, has differentiated itself almost entirely from philosophy, and become a 'natural' science. As a natural science, it is interested in the structure of the mental life — the characteristics of the elementary processes, and the laws of their combination — and not primarily in the function which ideas play in giving us knowledge. It is clear that psychology does not undertake to give a final account of all that mind is and does. It belongs to logic to investigate intelligence as a knowing function, just as it is the task of ethics to deal with the practical or active mental functions.

The practical question still remains as to whether this side of logic can be made profitable to students who have had no previous philosophical training. I am well aware of the difficulty of the subject, but my own experience leads me to believe that the main conceptions of modern logical theory can be rendered intelligible even to elementary classes. Of the incompleteness and shortcomings of my treatment I am quite conscious; but I have endeavored to make the matter as simple and concrete as possible, and to illustrate it by means of familiar facts of experience.

* * * * * * * * * *

J. E. C.

Cornell University,
August, 1898.

PREFACE TO THE THIRD EDITION

The present edition represents a somewhat thorough revision of this book, which had remained substantially unchanged since its first publication, eleven years ago. . . . The chapter with which the text formerly closed, "Rational and Empirical Theories", has been replaced by one entitled "The Unification of Knowledge". It has seemed important to conclude the discussion of the nature of thought with some statement of the meaning and function of the main categories which experience involves, and, in this connection, to indicate in a general way the necessity of a philosophical interpretation of the results of the special sciences.

The two purposes of an introductory course in logic which were emphasized in the preface to the first edition — to afford discipline in thinking and to furnish an introduction to philosophical studies — have thus been kept in mind in the present revision. The Third Part of the book presents an elementary account of knowledge from the developmental standpoint. The conceptions there treated in a somewhat systematic way are however introduced from time to time in the earlier chapters to modify and interpret the results of the older logical theories.

*   *   *   *   *   *   *   *   *   *

J. E. C.

CORNELL UNIVERSITY,
August, 1909.

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INTRODUCTION

CHAPTER II

IMPORTANT STAGES IN THE DEVELOPMENT OF LOGIC

§ 5. **Socrates and the Concept.** — Logic was founded as a separate and independent branch of inquiry by Aristotle (387–322 B.C.). Almost from the beginning of philosophical speculation, — which took its rise in the sixth century in the Greek cities on the coast of Asia Minor, and in Sicily and southern Italy, — questions had, however, been raised regarding the nature of knowledge and the proper value to be assigned to different forms of experience. More particularly these early thinkers emphasized the distinction between the knowledge given by sense-perception and that obtained by thinking or reasoning. The latter kind of knowledge, it was generally agreed, is alone trustworthy and genuine; while the senses, on the other hand, are bad witnesses and do not show us the true nature of things. One had only to reflect on his common, everyday experiences in order to vindicate some such distinction. The usual example cited as an illustration of this point is that of the twig which we *know* (by reasoning or otherwise) to be straight, but which *appears* to be bent when partly immersed in water.

In these early schools, however, logical questions about truth and knowledge were largely incidental, the fundamental interest being to explain the nature of the physical

universe. It was not until after the Persian wars, when Athens had become the intellectual and commercial centre of Greece, that the inner world of human experience — man's knowledge, moral beliefs and practices, customs, laws, and religions — came to be of primary interest and importance to philosophical inquirers.

The political prominence and wealth that came to Athens as a result of her leadership in the wars with Persia, led to the rapid transformation of the outward appearance of the city and also of the life and thought of its inhabitants. The new times and the wider circle of political and social activities thus opened up to citizens of Athens demanded that the older system of education — the traditional music and gymnastic — should be supplemented by some more advanced instruction. And in response to this demand there arose a class of teachers called Sophists, who made it their business to instruct young men in all the practical affairs of life, and especially in the use of words and the art of public speaking, or rhetoric, as it was called. The Sophists do not seem to have made it their object to teach truth to their pupils, or to inculcate in them a love and reverence for truth; they sought rather to make those whom they taught clever men of the world. In teaching the art of argumentation or public speaking they did not confine themselves to pointing out the methods by which true conclusions could be reached, but went on to teach the arts by which the judges could be persuaded, and tricks for the discomfiture of one's adversary. The rhetoric of the Sophists, in other words, was not a science of reasoning, but an art of persua-

sion and of controversy. It was not essential to have any real knowledge of the subject under discussion in order to argue well, from their point of view, but only to be well versed in all the arts of persuasion, and quick to take advantage of an opponent's errors.

The theory on which the teaching of the Sophists was based is usually known as Scepticism. The Sophists, that is, had come to the conclusion that it is impossible to find any fixed standard of truth. Looking at the diversity of individual opinions and of individual feelings, they declared that knowledge or truth as something objective, or the same for all, is an illusion. Only individual opinions exist; there is no standard by reference to which these opinions may be measured. Indeed, the words 'truth' and 'falsehood' can have only a practical meaning; each individual must be the measure of truth for himself. They lacked the scientific spirit that aims at truth which is objective and real; like men everywhere whose interest is exclusively practical, they thought truth in this sense abstract and unmeaning, and aimed only at knowledge which has some direct application.

Moreover, in the opinion of the Sophists, the same state of things exists with regard to our moral ideas. There is no standard of right and wrong, just as there is no standard of truth and falsehood. Each man has the right to choose what he regards as most advantageous for himself. The traditional rules of morality have no authority over the individual, nor is it possible to discover any rules of morality which are binding on all men. It is the part of wisdom

to consult one's own interest in acting, and to seek to secure one's own advantage. Moral distinctions, like logical distinctions, are purely relative and individual.

Socrates was the great opponent of this doctrine of Scepticism and Relativity as taught by the Sophists. They had concluded, from the diversity of individual opinion on moral questions, that there is no real or absolute distinction between right and wrong, false or true. Socrates, however, was convinced that if one examined more carefully the nature of the judgments which are passed by different individuals, one would find common elements or ideas. It is possible, he believed, to find a definite standard, both in matters of theory and in matters of practice. This common element, however, is not to be discovered in sensation, or in feelings of pleasure and pain; these experiences are purely individual, and can never serve as a universal standard. But beneath the diversity of sensations and feelings there is the thought, or *concept*, common to all men. When rational beings come to understand one another, they must agree as to the nature of the fundamental virtues, — justice, temperance, courage, etc. It is true that few men have thought about these matters, and are able to express their meaning clearly, but every man, as a rational being, carries these fundamental notions in his mind. Now in order to refute the moral scepticism of the Sophists (and it was this side of their teaching which Socrates especially opposed), it is necessary that the ethical notions, or concepts, implicit in the minds of men, shall be drawn out and carefully defined. How is this to be accomplished?

Socrates did not undertake to teach men what ideas they should hold regarding the nature of any of the virtues; he rather made them partners in an investigation, and by means of skilful questions tried to assist them in discovering the real nature of goodness for themselves. Another point to be noticed is that the definition of the various virtues was reached as a result of comparing the views of a number of individuals. In this way, by comparing the opinions of many men of different professions and of different grades of society, he was able to distinguish what was merely individual and relative in these opinions, and so to approach nearer and nearer to a true or generally satisfactory definition. But such a primarily negative process of gradual elimination of the false or imperfect definition is obviously very defective as a general method. As has often been remarked, it is much like trying to find a thing by first seeking to fix upon all the places where it is not.

Now Plato, the disciple of Socrates, improved upon the work of his master in two respects. In the first place he did not confine his attention wholly to moral conceptions, but showed that the Socratic method could also be used to refute the intellectual scepticism of the Sophists. In other words, he proved that in the concept, or thought, as opposed to sensation, a standard of truth is to be found, as well as a standard of morality. Knowledge arises from thinking, and it is possible to compare our thoughts, and thus reach what is objective and real in itself, however impossible it may be to find any basis of comparison in our sensations. In Plato's *Dialogues* a great many logical ques-

tions are raised, and in these discussions we can often see some of the fundamental distinctions of present day thought and language, as it were, in the making.

In the second place Plato developed a more positive method of definition by means of division. "The thing to be defined or classified is first referred to its genus, and then, by a series of dichotomies, the genus is divided into species and sub-species. At each division we ask to which of the species it gives us the thing to be defined belongs, and that is divided once more, the 'left-hand' species being left undivided as irrelevant to our purpose. The definition is found by adding together all the species 'on the right-hand side'."[1] "For instance, we may take an ancient, but not necessarily a good, definition of man. Man comes under the genus (corporeal) being. Corporeal beings may be divided into non-animal and animal. Animals again may be divided into non-rational and rational. Man is rational. Thus we obtain the definition: 'Man is the rational, animal, corporeal being'. Each part of the definition is wider than the thing defined; but the whole definition must be exactly equivalent to it."[2]

§ 6. **Aristotle and the Syllogism.** — Plato's method thus introduces a considerable amount of orderliness into the process of definition, but it is still far from complete or perfect. In fact he made no attempt to organize and arrange his results in a strictly scientific manner. To put the fundamental objection in the words of his famous

[1] Burnet, *Greek Philosophy*, Part I, p. 220.

[2] Latta and Macbeath, *The Elements of Logic*, p. 163.

pupil, Aristotle, Plato simply showed how things could, as a matter of fact, be classified and so far systematized. But he failed to demonstrate *the reason why* things should be organized in just one certain fashion and no other. Thus, in the example cited above, it appears that man may be classified as a corporeal being; and again, more definitely, as a rational animal. What is not shown is how or why men should be classified in precisely this way, or why the class of animals comes to be included in the class of corporeal beings.

We must go on, then, to try to find some 'middle term', as Aristotle called it, some connecting link, present alike, for example, in the nature of animals and of men. In this particular case Aristotle found that "having sensation and the power of independent locomotion" was common to both, and hence he could argue as follows:

All beings having sensation and the power of independent locomotion are animals,

All men are beings having sensation. . . .

Therefore, all men are animals.

This bit of reasoning not only allows us to include men in the class of animals; it also enables us to see why we should do so. Generalizing from such cases as this, Aristotle maintained that all genuine knowledge could be demonstrated in this way; in other words, that the **Syllogism,** as he called it, was the form of all valid reasoning. Aristotle thus became the founder of logic, as well as of many of the other sciences which have come down to us from the ancient world. His most important logical works are the

Categories, *De Interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and the *Sophistical Elenchus*, a treatise on Fallacies. These writings came afterwards to be known as the *Organon* (or scientific instrument) of Aristotle. They contained in the first place what we call theory of knowledge — a discussion of the structure of knowledge, and of the scientific principles upon which it rests. But they also furnished the practical application of these principles. In his doctrine of the syllogism, which is found mainly in the *Prior Analytics*, he showed (as we have just indicated) what he regarded as the only valid forms of reasoning from general propositions, and thus he sought to furnish the pattern or type to which all such proofs must conform. He also classified, in his work on Fallacies, the various species of false reasoning, and showed how false arguments could be refuted and exposed by the principles he had discovered. Here he indicated clearly the various ways in which certain kinds of propositions could be combined as premises to yield valid conclusions, and thought he had proved that no conclusion could be drawn from other combinations. This part of the Aristotelian logic has come down to us almost unchanged, and is the main subject of Part I of the present volume.

It will be noticed that in the doctrine of the syllogism Aristotle was dealing with that kind of reasoning which undertakes to *demonstrate* the truth of some fact by showing its relation to a general principle which every one admits. Hence this part of his work may be called the logic of proof or demonstration. Aristotle was at one time of

his life a teacher of rhetoric, and he seemed always to have aimed at putting this art of reasoning on a scientific basis. That is, for the rules of thumb and questionable artifices of the Sophists he wished to substitute general laws and methods of procedure based upon a study of the principles and operations of reason. By complying with the rules he laid down, an argument will necessarily gain the assent of every rational being.

But we do not employ our reason merely in order to demonstrate to ourselves or to others what we already know. We seek to discover new facts and truths by its aid. In other words, we not only wish to prove what is already known, but also to discover new facts, and we need a logic of Discovery, as well as a logic of Proof. This distinction between proof and discovery corresponds in general to that between Deduction and Induction. It is not an absolute distinction, as will appear later, for both processes are constantly employed in conjunction. But for the present it may be said that deduction is the process of showing how particular facts follow from some general principle which everybody admits, while induction shows the methods by which general laws are obtained from observation of particular facts. Now Aristotle, as we have seen, furnished a fairly complete theory of deduction, or method of proof. But he did not treat of induction, or the method of passing from particular facts to general laws, with anything like the same completeness. Moreover, what he did write on this subject received no attention for many centuries. Aristotle was himself a great scientific observer, and may

well be regarded as the father of many of our modern sciences. But in his logical writings his main object seems to have been to present a true theory of argumentation, as opposed to the false theories of the Sophists. Science, too, was only in its beginning when Aristotle wrote, and it was impossible for him to foretell the methods of discovery which it has actually employed.

After Aristotle's death (322 B.C.), and after the loss of Athenian independence, there was a great decline of interest in matters of mere theory which had no direct application to the practical affairs of life. The Stoic school did make some slight additions to logical theory, but like their opponents, the Epicureans, they regarded practice, the art of living well, as the supreme wisdom of life. The Romans, who derived their knowledge of Greek philosophy largely from the Stoics, were also interested in the practical advantages of logic, rather than in its theoretical side. It was the possibility of applying the laws of logic to rhetoric and public speaking that especially interested Cicero, who was the first to make Latin paraphrases and adaptations of Greek logic in his rhetorical works.

For more than seven hundred years, during the Middle Ages, the Greek language and literature were almost unknown in Western Europe. During this time almost the only sources of information regarding logic were Latin translations of Aristotle's *Categories*, and of an Introduction to the same work by Porphyry, who lived 232–303 A.D. Both of these translations were made by Boethius (470–525), who is best known as the author of *The Consolations of*

Philosophy. Even when scholars again became acquainted with the original works of Aristotle, in the latter part of the Middle Ages, they hardly understood their true significance. Nevertheless, a great deal of ingenuity was shown in subdividing and analyzing all possible kinds of argument, and giving the particular rule for each case. This process of making distinctions was sometimes carried so far that scholastic logic became extremely cumbersome and artificial.

It is not very difficult to understand why this set of logical rules seemed so satisfactory to the age of Scholasticism. The primary object of the men of this age was to weave the knowledge already possessed into a system, to show the connection and interdependence of all its parts, and thus to put it beyond the possibility of attack. And for this purpose the school logic seemed admirably adapted; it was always possible to bring every case which could arise under one or other of its rules.

There is no doubt that the Aristotelian logic had a real value of its own, and that it exercised a very important influence upon Western civilization, even in the form in which it was taught by the Schoolmen; but of course there is nothing complete or final about it. Its main purpose, as we have already seen, was to furnish a method by means of which the knowledge we already possess may be so arranged as to be absolutely convincing. But the centre of intellectual interest has changed since mediæval times. We are not content merely to exhibit the certainty and demonstrative character of the knowledge we already have, but we feel that there is much knowledge of importance

still to be discovered. So that in modern times, one may say, the desire to make discoveries, and so add to the general stock of knowledge, has taken the place of the mediæval ideal of showing that the traditional doctrines taught by the church are absolutely certain and convincing. And when men became conscious of the importance of gaining new knowledge, and especially knowledge about nature, they at once saw the necessity for a new logic, or doctrine of method, to aid them in the undertaking.

§ 7. **Bacon and the Inductive Method.** — All the great thinkers of the sixteenth and seventeenth centuries saw clearly that the school logic is simply a method of showing the certainty of the knowledge we already possess, and does not aid us at all in making new discoveries. A new method, they all declared, was an absolute necessity. The new point of view was put most clearly and eloquently by the famous Francis Bacon (1561–1626), at one time Lord Chancellor of England. Bacon called his work on logic the *Novum Organum*, thus contrasting it with the *Organon*, or logical treatises of Aristotle. An alternative title of the work is, *True Suggestions for the Interpretation of Nature*. Bacon begins this work by showing the advantages to be gained from a knowledge of nature. It is man's true business, he tells us, to be the minister and interpreter of nature, for it is only by becoming acquainted with the laws of nature that we are ever able to take advantage of them for our own ends. "Knowledge and human power are synonymous, since ignorance of the cause prevents us from taking advantage of the effect." The dis-

covery of the laws of nature, which is therefore of so great practical importance, cannot be left to chance, but must be guided by a scientific method. And it is such a method which Bacon endeavors to supply in the *Novum Organum.*

The method proposed by Bacon seems to us very simple. If we would gain new knowledge regarding nature, he says, and regarding natural laws, we must go to nature herself and observe her ways of acting. Facts about nature cannot be discovered from logical propositions, or from syllogisms; if we would know the law of any class of phenomena, we must observe the particular facts carefully and systematically. It will often be necessary, also, to put pointed questions to nature by such experiments as will force her to give us the information we want. Knowledge, then, must begin with observation of particular facts; and only after we have made a great number of particular observations, and have carefully classified and arranged them, taking account of all the negative cases, are we able to discover in them the general law. No hypotheses or guesses are to be made; but we must wait until the tabulations of the particular phenomena reveal the general 'form' or principle belonging to them all.

It will be frequently necessary to refer to Bacon's work in what follows. At present it is sufficient to note how Bacon showed that a knowledge of nature cannot be attained through general propositions and logical arguments, but that it is necessary to begin with the observation of particular facts. He emphasized, also, the importance of systematic observation and carefully planned experiments,

and showed that knowledge must begin with facts of perception. This is the method of induction, and Bacon is usually said to have been the founder of the inductive sciences.

Another and quite different method of extending knowledge was proposed by the great Frenchman, Descartes (1596–1650), who took mathematics as the type to which all knowledge should conform. That is, he supposed that the true method of extending knowledge was to begin with general principles, whose truth could not be doubted, and to reason from them to the necessary character of particular facts. Descartes and his followers thought that it was possible to discover certain universal propositions from which all truth could be derived through reason. They thus emphasized deduction rather than induction, and reasoning rather than observation and experiment. The spirit of Bacon's teaching was, however, continued in England by John Locke, in the *Essay Concerning Human Understanding* (1690). During the next centuries, philosophical thinkers were divided into two great schools: Rationalists, or those who agreed in the main with Descartes; and Empiricists, or Sensationalists, who followed the teachings of Bacon and Locke.

Although the natural sciences made great advances during the seventeenth and eighteenth centuries, there seems to have been no effort made to analyze and describe the methods which were actually being employed. In England, at least, it seems to have been assumed that all discoveries were made by the use of the rules and methods of Bacon.

One of the first writers to attempt to explain the method used by the natural sciences was Sir John Herschel (1792–1871). His work, *Discourse on the Study of Natural Philosophy*, was published in 1832. A little later, and with the same object in view, William Whewell (1794–1866), afterwards Master of Trinity College, Cambridge, undertook his *History of the Inductive Sciences*, followed some time after by the *Philosophy of Inductive Sciences*. The man, however, who did most towards putting the study of logic on a new basis was John Stuart Mill (1806–1873), the first edition of whose *Logic* appeared in 1843. We shall have frequent occasion to refer to this work in future discussions. It is sufficient to say here that Mill continues the empirical tradition of the earlier English writers in his general philosophical position. Mill's book gave a great impulse to the study of logic. Before it was published writers on the subject had confined their attention almost exclusively to syllogistic or deductive reasoning. Mill, however, emphasized strongly the importance of induction; indeed he regarded induction as the only means of arriving at new truth, the syllogism being merely a means of systematizing and arranging what we already know. Though few logicians of the present day adopt this extreme view, the importance of inductive methods of reasoning, and the necessity of studying them, have now become generally recognized. Most modern writers on logic devote a considerable amount of attention to induction. The reader will find that Part II of the present volume deals with this subject.

§ 8. **Logic from the Evolutionary Standpoint.** — Mill, however, like Bacon and Descartes before him, was still more or less under the dominance of the scholastic idea that the problem of logic is to supply a set of infallible rules by which the processes of reasoning might be guided and controlled in such a way as to ensure beforehand the absolute validity of the conclusions. We have already had occasion to comment on this conception of the function of logic, which is in effect to reduce it to a practical art of reasoning, rather than to think of it as a science of the forms and structure of thought in the modern sense of the word. The conception of 'modern' logic, worked out in the English-speaking world since the time of Mill, is a direct continuation of a movement started in Germany, by Immanuel Kant, about a hundred and fifty years ago. The fundamental idea upon which this logic is based, namely development, was first clearly formulated by Hegel, in his *Science of Logic* (1816–1818). Hegel's contribution, however, was rendered unattractive to most students of the subject because of the formalism and paradoxical mode of expression in which he clothed his thought. Fortunately, therefore, the work of Darwin in biology and the rapid extension of the evolutionary method to other fields, served to render the older idea of development more concrete and attractive. From this evolutionary standpoint logic seeks to describe and explain intelligence in terms of its own development. And it looks at the logical mind as a system of functions or activities that have a work to do and that progressively grow in the capacity to perform that work.

What this really means for the science of logic we hope to make quite clear in what follows. But perhaps we may add a few words here, in order to contrast this more concrete and distinctly modern conception of the science with the older view, which Kant asserted had persisted unchanged for two thousand years — *i.e.*, from the time of Aristotle to his own day.

The Aristotelian doctrine of the syllogism, at least as interpreted by later thinkers, is a purely formal science. In the form in which it is represented in ordinary text-books, it might perhaps be more properly described as the art of arranging our knowledge in such a way as to compel assent. The 'matter' with which thought is supposed to work is supplied to it in the form of concepts and judgments. The problem formal logic has to solve is to define and classify the various kinds of concepts with which thought operates, and to determine the various relations in which these stand when combined into judgments. Similarly it has to show what combinations of judgments can be employed as premises leading to valid conclusions in the syllogism. The criterion of truth employed in these investigations is the principle of non-contradiction or *consistency*. That is, inconsistent combinations of concepts are ruled out; but so far as the doctrine of the syllogism goes anything is true which is not self-contradictory.

Now without questioning the practical value of its canons, it is obvious that formal or syllogistic logic does not take any account of many of the processes of everyday thought, and that its rules go but a little way in helping us to dis-

tinguish the true from the false. For in the first place to think is not merely mechanically to combine and arrange ideas already in our possession. At best, this might enable us to render clearer and more definite what we already know, but would never enable us to gain new knowledge. The real movement of thought — as opposed to its merely formal procedure — consists in the formation of new ideas and new knowledge through actual contact with the world of experience.

The recognition of the importance of induction, and of the necessity of studying the methods of the inductive sciences, brought about by Whewell, Mill, and others, was a step in the right direction, for it called attention to a kind of thinking that occupies a large place in our intellectual life, and also gave rise to a truer conception of the nature of thought itself. But as we have seen, even Mill did not reach the idea which guides modern logicians, namely, that thought or intelligence, as the function of interpreting reality, is one from beginning to end; and that the various logical operations are all parts of one whole, or rather, are ways in which intelligence operates in different circumstances, or at different stages of its development. He still tended to treat of logical processes, like conception, judgment, and reasoning, as if they were separate and distinct processes, each existing, as it were, on its own account. In short, we may say that Mill was still influenced by an atomistic and static view of mind: he did not think of knowledge as essentially all of a piece, or of its movement or history as revealing its nature.

As opposed to the conception of mind as made up of separate ideas, the thought dominating modern logic is that of the unity and continuity of all intellectual life. Thought is regarded as an organic, living function or activity, which remains identical with itself throughout all its developing forms and phases. The problem, accordingly, that logic must set before itself is to show the unity and interrelation of all of the intellectual processes. No one of the steps or stages in this process can be completely understood when viewed by itself: each is what it is only in and through its connection with the whole of which it forms a part. No hard-and-fast boundary lines are to be drawn between the different stages of the reasoning process, but it must be shown that the whole nature of intelligence is involved more or less explicitly at each step. So, for example, the modern contention is that deduction and induction are two inseparable, though distinguishable aspects of scientific reasoning. Because they are distinguishable it is no doubt legitimate enough, for purposes of exposition, to treat of them separately, just as the physiologist studies one organ of the body at a time. But like the physiologist the logician must ultimately endeavor to grasp all the aspects of his subject in their concrete unity of structure and functioning. Furthermore, most logicians have come to recognize that both Aristotle and Mill had oversimplified views of deduction and induction respectively. It would, indeed, be contrary to universal experience in the case of every other science, that its pioneers should have comprehended their subject in its full complexity at the very

beginning. As we shall try to show in the proper place, the syllogism no more represents the complete nature of the deductive aspect of inference than an amœba represents the complete nature of animal life. And the same is true of Mill's famous Methods of Induction. Nevertheless we can only hope to understand the more complex in terms of the more simple; and this is sufficient warrant for beginning, as we do, with the syllogism, or the amœba, as the case may be.

BIBLIOGRAPHICAL NOTE

Besides the ordinary histories of philosophy the reader may consult for the history of logic: Prantl, *Geschichte der Logik im Abendlande*, 4 vols., Leipzig, 1855–1870; 2nd ed., 1927; which extends, however, only to the close of the mediæval period; Ueberweg, *System der Logik*, 4th ed., 1874; Eng. trans. of 3rd ed., London, 1874; Adamson, *A Short History of Logic*, London, 1911. Sir William Hamilton's *Lectures on Logic* and Cook Wilson's *Statement and Inference*, 2 vols., Oxford, 1926, also contain much historical information.

Among modern works on logic, the following may be mentioned: J. S. Mill, *A System of Logic*, London, 1st ed., 1843; 9th ed., 1875. W. S. Jevons, *The Principles of Science*, London, 1874; 2nd ed., 1877. H. Lotze, *Logik*, 1874; Eng. trans., London, 1881–1888. C. Sigwart, *Logik*, 2nd ed., 1889–1893; Eng. trans., London, 1895.

The newer development of logic is well represented by F. H. Bradley, *The Principles of Logic*, London, 1886; 2nd ed., 1922. B. Bosanquet, *Logic, or the Morphology of Knowledge*, London, 1888; 2nd ed., 1911; and *Implication and Linear Inference*, London, 1920. J. N. Keynes, *Formal Logic*, 4th ed., London, 1928. W. E. Johnson, *Logic*, 4 vols., of which three have so far been published, Cambridge, 1921— H. W. B. Joseph, *An Introduction to Logic*, 2nd ed., Oxford, 1916. L. S. Stebbing, *A Modern Introduction to Logic*, New York, 1931. C. I. Lewis, *A Survey of Symbolic Logic*, Berkeley, Calif., 1918. John Dewey, *How We Think*, Boston, 1910; and *Essays in Experimental Logic*, Chicago, 1918.

EXERCISES (II)

1. Can you suggest any reason why attention should have been directed first to the physical universe, rather than to problems of knowledge, morals, etc.?

2. Read Plato's Dialogue, "Protagoras", for a good description of the Sophists and of Socrates' relation to them.

3. What was the problem of knowledge that Socrates tried to solve, and how did he go about the solution of it?

4. What are Scepticism and Dogmatism, with respect to knowledge? How would you define the position of Socrates?

5. Why was it not possible for Aristotle to lay down a complete theory of inductive reasoning?

6. What is suggested, as to the validity of the doctrines of either thinker, by the opposition between Bacon and Descartes?

7. Describe Mill's services to Logic, and also the defects in his view of experience.

8. What do you understand to be the standpoint of Modern Logic, as contrasted with the earlier standpoints?

PART I

THE SYLLOGISM

CHAPTER III

THE SYLLOGISM AND ITS PARTS

§ 9. **The Nature of the Syllogism.** — The theory of the syllogism, as has been already stated, was first worked out by Aristotle. And it stands to-day in almost the same form in which he left it. A few additions have been made at different points, but these do not affect materially the main doctrine. In dealing with the nature of this type of reasoning we shall first try to understand its general aim and purpose, or the results it seeks to bring about. We shall then have to analyze it into the parts of which it is composed, and to examine and classify the nature of these elements. Finally it will be necessary to discover what rules must be observed in order to obtain valid conclusions, and to point out the conditions which most commonly give rise to error or fallacy.

In the first place it is to be noticed that syllogistic logic deals with the results of thinking rather than with the nature of the thought-process. Its object is less to give an account of the way in which thinking goes on, than to show how the ideas and thoughts we already possess may be combined so as to lead to conclusions that are certain, and that will compel assent. The ideas the syllogism uses as material are fixed by having been expressed in language. Indeed it is largely with words, as the expression of thoughts,

that syllogistic logic deals. Many of the discussions with which it is occupied have reference to the proper interpretation of words and propositions; and the rules it furnishes may be taken as directions for putting together propositions in such a way as to lead to a valid conclusion. Nevertheless it is important to remember that these rules are not arbitrary and external, but find their justification in the nature of thought. Indeed the theory of the syllogism, when rightly understood, may be said to reveal the fundamental characteristics of the process of intelligence. For it brings together facts in such a way as to make evident their interrelation and dependence. It connects a judgment with the grounds or reasons supporting it, and is thus a process of systematization. In order to understand the significance of the rules of syllogistic logic, then, it will generally be necessary to look beyond words and propositions to the act of thought whose results they express.

A great deal has been written regarding the principles or Laws of Thought which are employed in all logical reasoning. It seems better, however, to postpone the definite consideration of this subject until the student has learned more about the various operations of thought, and has had some practice in working examples. In dealing with the nature and principles of thought, in the third part of this book, it will be necessary to discuss this question at length. Even at the present stage of our inquiry, however, it is important to notice that syllogistic reasoning presupposes certain simple and fundamental principles of thought as the basis of its valid procedure. In particular, the regular syllogism is

founded on a principle called the law of Identity or the law of Non-Contradiction, according as to whether it is stated affirmatively or negatively. Stated affirmatively this so-called 'law' simply expresses the fact that every term and idea that we use in our reasonings must remain what it is. A is A, or has the same value and meaning wherever employed. The law of Non-Contradiction expresses the same thing in negative language. A cannot be both B and not B. If any term is taken to be the same as another in one connection, it must always be taken to be so; if it is different, this relation must everywhere be maintained. The data or materials employed in the syllogism are ideas whose meanings are supposed to be permanently fixed and expressed in words which have been carefully defined. It would be impossible to reason, or to determine the relation of our ideas, if their meaning were to change without notice, or if the words by means of which they are expressed were used now in one sense and now in another. It is of course true that our ideas regarding the nature of things change from time to time. And as is evident from one's own experience, as well as from the history of language, a corresponding change takes place in the meaning of words. But the assumption upon which syllogistic (and indeed all) reasoning proceeds is that the ideas to be compared are fixed for the meantime, and that the words by which they are expressed are used in the same sense throughout the course of the argument. The laws of Identity and Non-Contradiction are, then, simply the expression, in positive and negative form respectively, *of the principle of consistency*.

The one fundamental postulate of all thought is that it must be consistent with itself.

We may, however, have formal consistency without having real truth. It is quite possible that all the requirements of the syllogism may be met without its conclusions being true of reality. In other words, an argument may be *formally* true, but really false. It is not difficult to understand why this may happen. Formal logic accepts without criticism the ideas and judgments it compares. These data are, of course, the product of previous acts of thinking. But in proceeding to arrange them in syllogistic form we do not inquire whether or not they are true, *i.e.*, adequate to express the nature of the things for which they stand. For the purposes of formal logic it is only essential that their meanings be clearly understood, and that these meanings be regarded as fixed and permanent.

§ 10. **The Parts of a Syllogism.** — The syllogism may be said to express a single comprehensive act of thought. We may define the reasoning expressed in a syllogism as a judgment so expanded as to exhibit the reasons by which it is supported. In the reasoning,

> The geranium has five pointed sepals,
> This plant has not five pointed sepals,
> Therefore it is not a geranium,

we may say that we have the judgment, 'this plant is not a geranium', supported by the propositions preceding it, and that the whole argument expresses a single thought, complete and self-sufficient. It is possible, however, even when one is dealing directly with the process of thinking,

to distinguish in it different subordinate steps, various stages which serve as resting-places in the course of its passage to the complete and comprehensive form represented by the syllogism. But it is usual in dealing with such reasoning to take a more external view of its nature, and to regard it primarily as made up of words and propositions.

In this sense a syllogism is divisible into parts, and may be said to be composed of three statements or propositions. In the example given above the two propositions standing first are called the **Premises,** since they furnish the grounds or reasons for the proposition standing last and known as the **Conclusion.** However it is not true that we always find the two premises and the conclusion arranged in this regular order in syllogistic arguments. Oftentimes the conclusion is given first. Frequently, too, one of the premises is not expressed, and has to be supplied in order to complete the argument. Thus the statement, 'he must be more than sixteen years of age, for he attends the university', is an incomplete syllogism. The conclusion, as will be readily seen, stands first. There is also only one premise expressed. To put this statement in regular syllogistic form we have to supply the missing premise and arrange it as follows: —

All students of the university are more than sixteen years of age,
He is a student of the university,
Therefore he is more than sixteen years of age.

When one of the premises or the conclusion is not expressed, the argument is called an **Enthymeme.** Such an argument is defective only in form: the missing premise

or conclusion is really present and operative in thought. It is of great importance to form the habit of making clear to oneself the premises by which any conclusion claims to be supported. In this way groundless assumptions are often brought to light, and the weakness of an argument exposed. Whenever words like 'therefore', 'for', 'because', 'it follows', etc., are used in their proper signification, it is possible to find an argument composed of two premises and a conclusion. But one must not allow oneself to be imposed upon by the mere words, but must insist on understanding exactly what are the premises in the case, and how the conclusion follows from them. Not only may some part of the argument be taken for granted, as a kind of tacit agreement but very often there is a considerable amount of repetition and illustration of the principles employed, without any attempt to bring these various statements into relation in a formal way as premises of a syllogism. To reduce such arguments to syllogistic form requires a certain amount of interpretation of the statements they contain, frequently involving both condensation and rearrangement. Such reduction of the usual extended form of arguments is usually necessary in order to bring out clearly their essential structure — the premises which are actually employed to carry the conclusion — and to estimate accurately their logical force and value. Take, for example, the following passage from Jonathan Edwards: —

> Why should we be afraid to let persons who are in an infinitely miserable condition know the truth, or bring them into the light for fear it should terrify them? It is light that

must convert them if they are ever to be converted. The ease, peace, and comfort which natural men enjoy have their foundation in darkness and blindness; therefore as that darkness vanishes and light comes in their peace vanishes and they are terrified. But that is no good argument why we should endeavor to hold their darkness that we may uphold their comfort.

This may be reduced to the form of two syllogisms somewhat as follows:—

(1)

The terror of sinners is what dispels their blindness,
Light is a terror to sinners,
Therefore light is what dispels their blindness.

(2)

What dispels blindness is really a benefit to sinners,
Light is what dispels their blindness,
Therefore light is a real benefit to sinners.

It is necessary to carry the division of a syllogism still farther. The propositions of which such syllogisms as those cited above are composed may be divided into two **Terms,** and a **Copula** or connecting link. The terms, which are the extremes of the proposition, are named the subject and the predicate. Thus in the proposition, 'the fields are covered with snow', 'the fields' is the subject, 'are', the copula, and 'covered with snow', the predicate. To reduce a proposition to the logical form in which it is most conveniently treated, it is necessary to express it in such a way that the two terms are united by some part of the verb 'to be', preferably 'is' or 'are'. Thus the sen-

tence, 'No plant can grow without light and heat' would be expressed as a logical proposition in the following or some similar form: 'No plant is an organism which can grow without light and heat'. 'Men have strong passions' may be written 'Men are beings having strong passions'. It is always well to reduce a sentence to some such form, by substituting for the verb of predication some part of the verb 'to be'.

The analysis of the syllogism gives us the divisions under which it is convenient to treat this part of logic. We shall accordingly deal (1) with Terms, (2) with Propositions, and (3) with the Syllogism as a whole.

These divisions, however, are only made for the sake of convenience in treatment. It must not be forgotten that a term is a part of a proposition. To understand the nature of a term it is necessary to consider the part it plays in the judgment that the proposition expresses. In other words, the *function* of the term, rather than the form of the word or words employed, must be considered. It is of course true that we naturally and commonly use certain word forms to express certain kinds of ideas, just as in the grammatical sentence the different 'parts of speech' — nouns, verbs, etc. — have each a definite and comparatively permanent function. But even in the sentence it is the part that the word in its grammatical function plays, rather than its form, that determines whether it is to be classified as a noun or an adjective, a preposition or a conjunction. In dealing separately with terms, as we propose to do in the next chapter, we shall be occupied to a

large extent with the *form of words* in which certain kinds of ideas are usually expressed. But as the same word or group of words may be used for different purposes, it will be necessary, in order to understand the meaning of terms, to refer frequently to the various ways in which they are used in a proposition.

The same difficulty exists when propositions are considered by themselves, the relation to the complete argument of which they form a part being thus ignored. In this case, however, the results of the isolation are not so apparent; for a proposition forms, in a certain sense, a whole by itself. It is the expression of a judgment which, as we shall see later, is the unitary process of thought. It has thus a significance of its own, as expressing a more or less complete and independent act of thought. Nevertheless this independence and completeness are only partial and relative. To interpret a proposition correctly and fully we must know its context. In order to become intelligible it must be brought into relation with the other propositions stating the grounds or reasons upon which it rests, or the conclusion it helps to support. The logical meaning of a proposition, therefore, depends upon its function in an argument, and in treating of propositions this fact must not be forgotten. To understand is to appreciate the context.

§ 11. **Perception, Conception, and Judgment.** — Before beginning our examination of the elements of the syllogism, it is necessary to define some terms that describe certain phases or modes of our knowledge. These are Per-

ception, Conception, and Judgment. Judgment is both the elementary and the universal form of knowing. It includes all the others and uses them as a means to its own end of attaining truth. It may be described as the interpreting activity of the mind. At all the stages of experience it is at work, construing things in terms of ideas or meanings, transforming old ideas in the light of new facts, in order to render them more definite and more consistent. Judgment is thus the form of the general intellectual activity. To know anything is to express it in terms of ideas, to qualify it in our thought as this or that, as belonging to a certain class of things, or perhaps as differing in some respect from another class of things. But it must not be supposed that judgment — or any form of thinking — is concerned only with *our own ideas.* Judgment is the interpreting, idealizing response of the mind to the real world, with which it is always in relation. To think is not to play with our own ideas; real thinking deals, more or less directly, with a world of real objects and persons. In the process of judgment, then, reality is interpreted and its meaning expressed in terms of ideas. The expression of such an act of thought is a proposition, which, as we have already seen, is very often composed of a subject and a predicate term related by means of a copula.

Now the terms of which a proposition is composed may be either percepts or concepts, *i.e.*, the result of a perceptive act or of a conception. A percept is the result of the mind's direct mode of apprehending real things as distinct individuals. Hence a percept always refers to 'this' or 'that',

some distinct individual thing having its own place in space or in time. Thus I perceive, or have a percept of, the objects in this room, and of the tree which I see through the window. Similarly one may perceive the particular states of consciousness in one's mind. A concept, on the other hand, is a general meaning or idea. It does not refer directly to some one object of sense. It is not an individual embodiment of a particular thing, but is a thought-construction, carrying with it the idea of a general nature or meaning that may apply to a number of individuals. Thus my direct experience of the individual tree at which I am looking is a percept, the general idea of tree that I use when I say 'trees are either deciduous or evergreen' is a concept. I may have a percept of the Statue of Liberty at the entrance to New York harbor; 'liberty', on the other hand, is a concept made up of a more or less definite group of meanings, unified and held together by the word in which it is expressed.

What, now, is the relation between the percepts and concepts expressed in the terms of a proposition, and the judgment represented by the proposition as a whole? In the first place, it is to be noted that percepts and concepts are the results of previous acts of judgment. Ideas are formed only through the mind's own act of interpretation; they never pass over into the mind from some external source as ready-made objects. Even in the case of perception, where the object seems to be thrust upon us, a little reflection will show that the judging activity of attention is involved, selecting and arranging the various sensation elements, and interpreting them as the parts of a single concrete object,

in accordance with past experience. A concept like 'man' or 'justice' is still more obviously a thought or judgment construction. As expressed in words, it may be said to be an embodiment of a judgment or a group of judgments.

And in the second place it is from these percepts and concepts that new judgments proceed. In other words, the basis of our thought in going on to the discovery of new facts and relations is *what we already know*. But what we already know at any time is summed up in the ideas we possess, that is, in the percepts and concepts which have been formed by previous acts of judgment and embodied in names. In the development of our knowledge, however, we are constantly discovering that our knowledge on this or that point is unsatisfactory. The old way of thinking is perhaps too vague and indefinite to furnish us with a satisfactory rule of action, or it may be perceived to be inconsistent with new facts that have arrested our attention. Indeed, the inadequacy of the habitual, accepted point of view may be forced upon us in a variety of ways. Frequently the occasion is furnished by some practical necessity of action. Necessity is oftentimes the mother of invention and the spur to the discovery of new theories and conceptions. In other cases the stimulus to criticize our old conceptions may come from social intercourse; the conflict of our views with those of people with whom we converse, or whose opinions we read, first arouses us from our dogmatic slumber. More rarely theoretical interest may be aroused without any external occasion, and the desire for truth and consistency may itself be sufficient to lead one to reëxamine

and transform one's old ideas. Whatever the stimulus, thinking is, on one side, a process in which old conceptions are recast and accepted truths transformed, a constant process of change in which the old conceptions are superseded and destroyed. The old terms, both percepts and concepts, forming the starting-point, are reconstituted through a new act of judgment. From one point of view it may be said that, like Kronos, thought exists by devouring its own children.

But there is another side. Thinking is a process of conservation as well as of transformation. The old ideas are not so much destroyed and displaced by the new judgment as further developed and defined. The partial truth which the old formulas contain is taken up and preserved in the later judgment or series of judgments. Then the results of these judgments are again laid down as new thought-contents embodied in language, and these in turn form the starting-point for further judgments. Hence the two aspects or moments of thought — what we have called the transforming and the conserving functions, — mutually presuppose and imply each other. They are not distinct and independent mental operations, but organically related moments or phases in the life of thought. Perceptions and conceptions can arise only through judgments, while judgments presuppose perceptions and conceptions as their necessary basis and starting-point. Thus the total movement of the whole thought-process is rightly described as judgment, since the growing insight of mind is its beginning and end.

EXERCISES (III)

1. What is a syllogism?

2. What is the principle upon which syllogistic reasoning depends? Is this principle capable of proof? Explain.

3. Distinguish between the function of the principles and that of the premises of a reasoning.

4. What is meant by calling logic a 'formal' study?

5. (*a*) Illustrate the fact that a term cannot be understood apart from the proposition in which it occurs. (*b*) Show how the same is true of a proposition relatively to the context to which it belongs.

6. Illustrate in a similar fashion the relation between perception, conception, and judgment.

7. Illustrate the fact that thought has both a transforming and a conserving function.

CHAPTER IV

THE VARIOUS KINDS OF TERMS AND RELATIONS

§ 12. **Words, Names, and Terms.** — A logical term, as we have already seen, is any word or group of words that may be used as the subject or predicate of a proposition. It is only in propositions, and as elements of propositions, that terms have any assignable meaning. It is impossible, therefore, to fix the meanings of isolated terms without reference to the way in which they are used in propositions. But before proceeding with this classification it may be as well to distinguish terms, as above defined, from both words and names, with which they have sometimes been confused.

Words, of course, are oral or written signs designed for purposes of communication. Certain words, such as nouns and adjectives, may function as terms in a proposition; other words, such as verbs, adverbs, prepositions, and conjunctions, usually do not. And the same is true of word-combinations such as phrases and clauses. For example, in the proposition, 'all normal men are rational', the phrase 'all normal men' is the subject term, while the adjective 'rational' is the predicate. In exceptional cases, however, words and combinations of words, not usually employed as such, may become subject or predicate terms, as, for example, in the proposition, 'of is a preposition'. In general, then, the distinction between words and terms is

easy enough, but that between that particular kind of words called names, and terms, is somewhat more difficult. Certain logicians, notably Jevons and Mill, following Hobbes, define names in such a way as to suggest that they may be used by themselves, apart from their functioning as subject or predicate in a proposition. And in so far as this is possible, names and terms are quite distinct. In a dictionary, *e.g.*, we find a list of names, many of which are capable of more than one meaning, these various meanings simply indicating the ways in which names may be used as terms. Thus the same name may stand for several different terms, according to the context — especially that part of the context which is the proposition — in which it is used. 'Dog', 'house', 'play', are instances of the numerous illustrations that come readily to mind in this connection. Punning is of course largely based upon this fact that names have various meanings.

§ 13. **Singular, General, and Collective Terms.** — The first division of terms we have to notice is that into Singular or Individual, General, and Collective.

(1) A **Singular** or **Individual** term is one that can be applied in the same sense to but a single thing. The main purpose of singular terms is to refer to, or identify, some thing or experience which can be regarded as a single existence. Proper names are all singular. It is true that proper names are sometimes used to denote a class of objects, as *e.g.*, 'a Daniel', 'a Mephistopheles'. But when thus employed they lose their real character as proper names. That is, their function is no longer merely to identify certain

individuals by *naming* them, but to *describe* them by mentioning certain qualities or characteristics that they are supposed to possess. But the ordinary purpose in using a proper name is to indicate some individual to whom the name belongs. In this sense, then, proper names are singular.

Likewise any word or group of words applied to a single thing may be regarded as singular. And by 'single thing' we mean anything thought of as one, as well as objects perceived through the senses. Thus 'the waterfall just below the bridge', 'the thought of the present moment', are singular terms, and so are words like 'justice', 'goodness', 'the chief end of man'. It is more doubtful whether we should call terms such as 'whiteness', 'sweetness', singular, since we speak of different degrees and kinds of whiteness and sweetness. The question would have to be decided in every case by reference to the way in which the terms are employed in propositions.

(2) A **General** term is a name capable of being applied to a whole group of objects. It is not limited, like the singular term, to a single thing, but can be used in the same sense of an indefinite number of units. All class names, like 'metal', 'man', 'works on logic', are of this character. Thus a general name is one that refers to a group which may be divided into smaller groups or into individual units. Iron, gold, silver, etc., are 'metals', and A, B, and C, 'men'.

A **Collective** term, on the other hand, is a name applied to a number of individual things when taken together and treated as a whole, as 'an army', 'an audience'. It is important to notice carefully when a term is collective and

when it is general. A general term is a name that applies equally to each individual of the group; or, in other words, it may be used of the individuals *distributively*. A collective term belongs to the whole, but not to the separate parts of the whole. Thus we say that 'soldier' is a general term and is used distributively of each man in a regiment. 'Regiment', however, is a collective term, for it applies only to the whole group, and not to the individual soldiers.

Ambiguity sometimes arises from the fact that the English word 'all' is used in both of these senses since it may mean 'all taken together' or 'each and every'. Thus we can say: 'All the angles of a triangle are less than two right angles', and 'All the angles of a triangle are equal to two right angles'. In the former sentence the word 'all' is used distributively, in the latter collectively. In Latin two different words are used: *cuncti* expresses the collective sense of 'all' and *omnes* its distributive signification.

It is worth repeating in this connection that it is the *use* which is made of terms, rather than the *form* of the words composing them, that determines their logical character. Thus terms which are collective in one connection may be general in another. 'Regiment', for example, is a collective term with reference to the soldiers composing it, but general when used as a common term for a number of similar divisions of an army. The same is true of terms like 'grove', 'mob', 'class', etc. Again, collective terms may be very properly regarded as singular when the proposition in which they are used emphasizes the unity and solidarity of the group. A proper name is sometimes ap-

plied to a collection of individuals that are permanently united or that have acted ogether on some historic occasion, as, for example, 'The Fifth Cavalry Regiment', 'The Charge of the Six Hundred'.

§ 14. **Abstract and Concrete Terms.** — Terms are further divided into those which are *abstract* and those which are *concrete*. The word 'abstract' is often used popularly to describe anything difficult to understand. Etymologically it signifies drawn off, separated (*abstraho*, to draw off, take away). We may distinguish two senses in which the word is used, both, however, being derived from its etymological signification.

(1) A term is called *abstract* when it refers to something that cannot be directly perceived through the senses, or otherwise directly experienced as an individual object or state, and *concrete* when such form of experience is possible. Thus 'a beech tree', 'a tall man', 'a sweet taste', being names of things that can be perceived, are concrete. Words like 'sweetness', 'hardness', etc., have no objects of immediate experience corresponding to them, and are for this reason called abstract. The same is true of terms like 'individuality', 'equality', 'justice', etc. These words represent objects of thought rather than objects that are directly experienced. There may be cases or instances of 'equality', 'justice', etc., which fall under our perception, but the real object to which these words correspond is not a thing which can be perceived through the senses at all. Their reality is conceptual, or for thought, not something directly revealed through the senses.

It is important to notice that there are degrees of abstractness in terms according as the objects for which they stand are nearer to, or farther removed from, ordinary sense-perception. All general or class names are abstract. One cannot point to a single object to which the term 'metal', for example, or the term 'man' corresponds. But although such terms have no direct sensuous object, we feel that they stand nearer to sense-perception and are therefore less abstract than words like 'animal', 'inorganic substance'. These terms, again, are perhaps less abstract than 'energy', or 'spirit', or even singular terms like 'justice', 'the ground of the universe', etc.

(2) Again, the word 'abstract' is applied to any object treated apart from the whole to which it belongs. Thus it would be an abstraction to study the nature of a leaf in complete isolation from the plant to which it belongs, or to consider the nature of a man without regard to the social institutions — family, church, state, etc. — of which he is a member. Of course it is essential when dealing with a complex whole to analyze it into its parts, and to understand just what is the nature of each part when taken by itself. But in order to comprehend fully the nature of the parts it is necessary to restore them to their proper setting, and to see their relation to the concrete whole. In this sense of the word, then, 'abstract' applies to what is taken out of its proper setting, broken off, and considered apart from the things to which it is organically related. Concrete, on the other hand, means what is whole and complete, a system of things mutually supporting and explaining one another.

Since science has to analyze things into their elements and to investigate and describe these elements in detail, it is impossible entirely to avoid abstraction. But it is necessary, in order completely to understand the nature of a complex object, that the abstractions of analysis shall be corrected. In other words, the concrete relations in which things stand must not be ignored in investigating them. The conception of evolution in recent times has done much to render the biological sciences more concrete in the sense in which we are now using the term. For it has substituted for the old method of treating each species of plant and animal as distinct and separate, 'cut off from each other as if by a hatchet', the view that all organic beings are members of one family and can be properly understood only in their relations to one another.

It is interesting to notice that from this point of view sense-perception is more abstract than thought. For the senses represent things in isolation from each other. Each thing is known in sense-perception as a separate individual, occupying its own space and time, and in this way cut off from its fellows. It is the business of thought, on the other hand, to discover the relations between things, and the principles according to which they are united. Thinking thus overcomes the abstract point of view of sense-perception by showing that what appear to the latter as separate objects are really closely and necessarily connected as members of a common unity or system. Each science takes as its province certain facts which resemble one another, but which nevertheless appear to sense-perception to be

quite independent. It attempts by thinking to bring these facts into relation, to show that they are all cases of some law, that there is a common principle which unites them as parts of a whole or system. The law of gravitation, for example, expresses the unity which thought has discovered in things that appear to sense-perception as different as the falling of an apple, the movements of the heavenly bodies, and the ebb and flow of the tides. Scientific knowledge, then, is more concrete than the facts we learn from ordinary sense-perception, because it brings to light real unity and connection in facts which appear to be entirely isolated and independent from the latter point of view.

In employing the terms 'abstract' and 'concrete' it is of the utmost importance to distinguish the two significations of the words. From one point of view, as we have seen, all thought terms are abstract, as opposed to words that refer directly to objects of sense-perception. In another sense 'abstract' denotes what is partial and incomplete, what is taken by itself and out of relation to the system of things to which it belongs. And since the real connection and relations of things are not given by perception but have to be discovered by thought, the knowledge which the latter yields is more concrete, in this latter sense of the term, than that afforded by the former.

§ 15. **Positive and Negative Terms.** — The distinction between positive and negative terms is obvious. **Positive** terms express the existence of some quality or group of qualities, in the objects they denote; as, 'happy', 'good', 'equality', 'organism', etc. A **Negative** term on the other

hand indicates the absence of qualities or properties in some object; 'bad', 'unhappy', 'inorganic', 'injustice' are negative terms. Negative terms are often formed from positive by means of the affix *less*, as in 'hopeless', or by means of certain prefixes, of which the more common are *un*, *in*, *dis*, *a*, *anti*. Words positive in form are, however, often negative in meaning, and are used as the contradictories of other terms. Thus 'ignorant' is generally regarded as the negative of 'learned', 'darkness' is the negative of 'light', etc. It is not always possible, however, to find a separate word to express the exact opposite of every positive term. Words are used primarily to express the presence of qualities, and the negative idea may not be referred to so frequently as to require a separate word to express it. Thus there is no independent term to express the opposite of 'transferable', but by employing 'non' as a negative prefix we obtain 'non-transferable'.

It is always advisable when we wish to limit a term strictly to its negative application to employ *not* or *non* as a prefix. Words negative in form frequently have a more or less definite positive signification. Terms like 'unhappy', 'immoral', do more than indicate the absence of qualities: they express some positive properties of the objects to which they are applied. We speak of a person 'being positively unhappy'; and we employ 'non-moral' to express the simple negative relation rather than 'immoral'.

On the other hand there are certain terms which are positive in form that express the absence of qualities or attributes. Words like 'blind', 'dumb', 'maimed', 'or-

phaned', may be given as examples. These are often called privative terms, rather than negative, the distinction being that they refer to qualities or attributes that the objects to which they are applied naturally and usually have, but of which they have been deprived, or that they have never possessed. Thus 'blind' implies loss or lack of the ability to see. Again, some terms seem to be positive and negative solely in relation to each other. 'Element' and 'compound' are related as negatives or contradictories. In such cases it is difficult to say which term is in itself negative or positive.

It is important to notice the distinction between the relation in which positive and negative terms stand to each other, and that expressed by words having to do with opposite extremes of something possessing quality or degree. Positive and negative terms are mutually **Contradictory.** An element is what is *not* a compound, 'dishonest' is the contradictory of 'honest', and as contradictories they have no middle ground between them. What is not an element is a non-element or a compound. **Contrary** terms, on the other hand, express a great difference of degree in the objects to which they refer, yet there is always middle ground between them. Thus 'foolish' is the contrary of 'wise', 'cold' of 'hot', and 'bitter' of 'sweet' yet we cannot say that a man must be either wise or foolish, a taste either sweet or bitter. The logical contradictory of 'wise' is 'not-wise', of 'bitter' is 'not-bitter', etc. Contrary terms, then, must be carefully distinguished from contradictories, and we cannot conclude because one con-

trary term is false in a given case that the other is necessarily true.

§ 16. **Absolute and Relative Terms.** — Another classification of terms usually given by logicians is that into absolute and relative terms. An **Absolute** term refers to an object existing by itself and has an intelligible meaning when taken alone. Thus 'tree', 'house', 'the State of New York', are examples of absolute terms. A **Relative** term, on the contrary, is a name which derives a meaning only from its relation to something else. The term 'parent', for example, cannot be thought of except in relation to 'child'. Similarly 'teacher' is relative to 'pupil' and 'cause' to 'effect'. Relative terms usually go in pairs and are known as **Correlatives.** Adjectives as well as nouns may be related in this way. The presence of one quality or characteristic in a thing frequently implies the presence of others. Thus ignorance and superstition, sympathy and tolerance, are necessary correlatives, because the one involves the other, or is invariably connected with it.

§ 17. **Extension and Intension of Terms.** — In the foregoing sections of this chapter we have explained the main distinctions exhibited by logical terms. It is now necessary to notice two different purposes for which terms are employed. In the first place terms are used to *refer* to things, to name and identify them. Thus 'man' refers to the different individual men, John Smith, Thomas Brown, etc., as well as to the various classes of men, Caucasians, Indians, Mongolians, etc. As denoting or naming objects, whether these be individual things or classes of things, terms are

said to be employed in Extension. But words are also used to *describe* as well as to name. That is, they represent the qualities or attributes belonging to things for which they stand. They are not bare names without signification; but, as the expression of ideas, they stand for certain qualities or characteristics that things are judged to possess. 'Man', for example, is not merely a name that may be applied to individual human beings or races of men; but it implies that the objects so named have certain qualities, such as animal life, reason, and the power of communicating with their fellows. When words are used in this way to define or describe things, rather than merely to name them, they are said to be employed in Intension.[1]

It is essential to accustom ourselves to distinguish these two functions or uses of a term, — to notice, that is, the things or classes of things to which it applies, and also to reflect upon the signification, or ways of judging about these things for which it stands. The Extension of a term, as has been said, indicates the objects to which a name applies, and the Intension the qualities or attributes which it signifies. From the point of view of extension, therefore, 'planet' may be defined by mentioning the names of the various planets, Mercury, Venus, the Earth, Mars, etc. Similarly a term like 'carnivora' might be given in ex-

[1] The terms 'Denotation' and 'Connotation' were used by Mill instead of Extension and Intension, respectively, and have been adopted pretty generally since his time. To 'denote' is to point out or specify the objects for which a term stands; and to 'connote' is to take account of the attributes or qualities which a name implies. The words 'depth' and 'range' are also sometimes used as synonymous with Extension, and 'breadth' or 'comprehension' instead of Intension. The terms to be remembered, however, are Extension or Denotation, and Intension or Connotation.

tension by naming seals, bears, weasels, dogs, wolves, cats, lions, etc. Usually, however, we define from the point of view of intension, that is, by stating the qualities or characteristics for which the term stands. Thus we give the intensive meaning of 'planet' as a heavenly body that revolves in an elliptical orbit around the sun. 'Carnivora', defined from the same point of view, are mammalian vertebrates which feed upon flesh. It is not unusual, however, to supplement an intensive definition by turning to extension and enumerating examples. Thus we might add to the definition of 'carnivora' just given the words, 'as lions, tigers, dogs', etc.

It is sometimes said that the intension and extension of terms vary inversely. This is simply an attempt to give a mathematical form of statement to the fact that the more a term is defined or limited by the addition of attributes, the fewer are the objects to which it applies. 'As the intension of a term is increased its extension is diminished, and *vice versa*', is the form in which the relation is often stated. For example, let us begin with some class name like 'animal', which has a great extension, and add a new attribute, 'rational'. We get 'rational animal' = man. This term now applies to a much smaller number of individuals than 'animal'. The extension of the former term has been diminished, that is, by increasing the intension. If we add to 'man' still another attribute like 'white', we again lessen the number of individuals to which the term applies. In general, then, it has been maintained that the extension of a term is lessened as it is made more definite by the addition

of new attributes. And conversely, by stripping off attributes, by 'decreasing the intension', the number of individuals to which a term applies may be said to be increased. There is, however, no exact ratio between the increase or decrease of intension and the corresponding change in extension. Indeed the extension of a class may increase greatly without any loss of intension on the part of the term by which the idea is expressed. Thus the meaning or intension of the term 'man' has not lost, but rather gained, during the last hundred years by the increase of population throughout the world.

In other words, intension and extension are in reality incommensurable. They are not calculable quantities such as those between which inverse ratios naturally obtain. To go no further, we cannot always say what one quality or attribute is. Common words like 'beautiful', 'good', 'rational', 'physical', represent not one but a variety of qualities which it is impossible to calculate. And if you first use a word in one context with one intension, and then in another context with another intension, you are really using two different terms.

Now extension and intension, according to the view just given, represent two different uses or functions of terms. Every term denotes some object or group of objects more or less directly, and at the same time connotes or signifies certain qualities or attributes. Sometimes the one purpose, sometimes the other, is predominant. Proper names, for example, are used primarily to denote or mark out things, and do not directly qualify or describe them. And on the

other hand, in the proposition, 'these animals are all vertebrates', the predicate term 'vertebrates' is employed less as a name of a number of animals than as a description of their qualities. Yet in both these cases the terms employed have the double function of naming or denoting objects and of connoting qualities.

Mill, however, and certain other logicians who follow him, seem to make an absolute distinction between connotative and non-connotative terms.

> A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute. By a subject is here meant anything which possesses attributes. Thus 'John', or 'London', or 'England', are names which signify a subject only. 'Whiteness', 'length', 'virtue', signify an attribute only. None of these names, therefore, are connotative. But 'white', 'long', 'virtuous', are connotative. The word 'white' denotes all white things, as snow, paper, the foam of the sea, etc., and implies or, in the language of the school men, *connotes* the attribute *whiteness*. . . . All concrete general names are connotative. The word 'man', for example, denotes Peter, James, John, and an indefinite number of other individuals, of whom, taken as a class, it is the name. But it is applied to them because they possess, and to signify that they possess, certain attributes.[1]

There are, then, according to Mill, some names which are denotative merely, and others which are connotative as well. But this thesis is robbed of much of its plausi-

[1] Mill, *Logic*, Bk. I, Ch. II, § 5.

bility when we remember that the subject under discussion is *terms*, rather than mere *names* as such. The latter have, as we have seen, no fixed and unvarying extension or intension at all. Mill, however, seems to think of a name as denoting a mere aggregate of things and as connoting a mere aggregate of qualities. He first takes names out of their context as terms in a proposition, and then ascribes them to 'subjects' or things, which may or may not 'possess' certain 'attributes'. Things and attributes thus seem to maintain an independent existence, and upon occasion to be more or less mechanically conjoined together. But if we stop to ask what a thing, what a 'subject' is, apart from its qualities, or what a quality is, apart from things, we find ourselves at a loss for an answer. Rather we have to think of a thing as a system of qualities or attributes, more or less closely interrelated and mutually determining one another. It is both a one and a many; has both a denotation and a connotation. And in the proposition this fact gets expression, in one way or another. A proposition, that is, is just a medium wherein the systematic interconnection of things with each other, through their respective attributes, is displayed.

§ 18. **The Classification of Relations.** — So far we have been considering terms as functioning in propositions which may be logically analyzed into a subject and a predicate — in which the relationship between the terms is that of predication. But as we shall see more fully in the following chapters, there are also very many propositions which it seems hardly natural to analyze in this way — in which

the relationship between the terms is not that of predication but should be described in some other fashion.

Leibniz (1646–1716) was one of the first specifically to call attention to this fact, although a thorough consideration of the way in which relative terms function in propositions might have led to its discovery at any time. The passage, in Leibniz's *Letters to Clarke*, in which he deals with this subject, is as follows: "The *ratio* or *proportion* between two lines L and M, may be conceived three several ways; as a ratio of the greater L to the lesser M; as a ratio of the lesser M to the greater L; and lastly, as something abstracted from both, that is, the ratio between L and M, without considering which is the antecedent or which the consequent; which the subject, and which the predicate."

Now it is this last way of analyzing such propositions that Leibniz himself preferred, and which has recently been developed by philosophers interested primarily in the logic of the mathematical sciences. Propositions like 'A is equal to B', 'L is greater than M', 'X is the brother of Y', according to these thinkers, do not contain a subject and a predicate, in the traditional sense of these words. The relationship between the relative terms A and B, L and M, X and Y, is manifestly quite different from that between terms referring to a thing and its qualities ('this book is heavy'), or that between two classes of things ('men are mortal beings').

Perhaps it may be said that in the subject-predicate type of proposition emphasis is laid upon the terms between

which the relation of predication holds, while in the other type emphasis is laid upon the relationship itself. In all cases, however, whether we have to do with one type of proposition or the other, the office that a proposition really performs is to bring about and give expression to a further determination of a given subject-matter already partially determined or understood. In some cases this subject-matter may be explicitly indicated by a single subject-term in the proposition; in other cases it is implied by, but not expressly formulated in the proposition as a whole.[1]

The full logical significance of this distinction between different types of propositions will be considered in the proper place. In this chapter we are concerned only with the fact that in the light of such a distinction of emphasis it occurred to certain thinkers that it was quite as possible and important to classify the various kinds of (non-predicative) relations, as the various kinds of terms, to be found in propositions. The simplest classification that has been proposed is that into Symmetrical, Asymmetrical, Transitive, and Intransitive relations.

A **Symmetrical** relation is one which holds between both A and B and B and A, as in the propositions: 'A *is equal to* B', and 'B *is equal to* A'. An **Asymmetrical** relation, on the other hand, is one which never holds between both A and B and B and A, as in the propositions: 'A *is greater than* B', and 'B *is less than* A'. A relation is said to be **Transitive** if, whenever it holds between A and B, and also between B and C, it also holds between A and C. Thus if

[1] See below, pp. 188 ff.

A *is before* B, and B *is before* C, A *is before* C. **Intransitive** relations, on the other hand, are such that, if A has the relation to B, and B to C, A never has it to C. If A *is the father of* B, and B *is the father of* C, A cannot be *the father of* C. In all cases the relation that holds between B and A, corresponding to that which holds between A and B, and so on, may be called its **Converse.** Thus, 'B = A' is the converse of 'A = B', and 'B is less than A' is the converse of 'A is greater than B'.

Closely associated as they obviously are with relative terms, a relation together with its converse might perhaps appropriately be called correlative expressions. At all events it must be borne in mind that any such classification of relations, like the similar classification of terms, has as its primary purpose the rendering clearer to us the precise sense in which words and combinations of words function as elements in propositions. Neither terms nor relations can have any logical significance by themselves, in abstraction from their natural context.

EXERCISES (IV)

1. Distinguish carefully between the following, giving original examples of each: (*a*) words, (*b*) names, (*c*) terms.

2. Write several propositions to illustrate (*a*) general, (*b*) collective, and (*c*) singular terms. Use the same word in different propositions to illustrate the distinction between general and collective terms; between collective and singular terms. *Do not use the examples given in the text.*

3. Discuss the nature and consider the logical value of the distinction between abstract and concrete terms. Which do you consider to be the most abstract of the sciences?

4. 'Logic is in one sense the most abstract, in another sense the most concrete, of the sciences'. Discuss.

5. (*a*) Strictly speaking there are no negative terms, for affirmation and denial are characteristics, not of terms but of propositions. Discuss this statement. (*b*) Under what conditions do we have to express our knowledge negatively?

6. Distinguish carefully between contradictory and contrary terms. Illustrate with original examples.

7. (*a*) In what sense, if in any, may we speak of increasing or decreasing the intension of a term? (*b*) Consider critically the view that some terms are denotative merely.

8. Illustrate, with examples of your own, the different types of relations expressed in propositions.

CHAPTER V

DEFINITION, DIVISION, AND CLASSIFICATION

§ 19. **Fixing the Meaning of Terms.** — We have already referred to the necessity of definitely fixing the meaning of the terms we employ in reasoning. In ordinary life words are frequently used in a loose and shifting way, without any clear conception of the qualities or properties they connote, or of the objects to which they apply. But logic demands that we shall have clear and precise ideas corresponding to our words, and that the signification and scope of the latter shall be carefully determined.

Bacon, Hobbes, Locke, Hume, and nearly all of the older philosophical writers have warned us against the abuse of words. The whole matter has been expressed very clearly by Locke, from whom we quote the following passage: —

> For he that shall well consider the errors and obscurity, the mistakes and confusion, that are spread in the world by an ill use of words, will find some reason to doubt whether language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there, that, when they would think on things, fix their thoughts only on words, especially when they would apply their minds to moral matters; and who, then, can wonder if the result of such contemplations and reasonings about little more than sounds, whilst the ideas they annex to

them are very confused and very unsteady, or perhaps none at all; who can wonder, I say, that such thoughts and reasonings end in nothing but obscurity and mistake, without any clear judgment or knowledge?

This inconvenience in an ill use of words men suffer in their own private meditations; but much more manifest are the discords which follow from it in conversation, discourse, and arguments with others. For language being the great conduit whereby men convey their discoveries, reasonings, and knowledge, from one to another; he that makes an ill use of it, though he does not corrupt the fountains of knowledge, which are in things themselves, yet he does, as much as in him lies, break or stop the pipes whereby it is distributed to the public use and advantage of mankind.[1]

The remedy for the obscurity and confusion in language is to be found in clear and distinct ideas. We must endeavor to go behind the words and realize clearly and distinctly in consciousness the ideas for which they stand. Now the means that logic recommends for the attainment of this end is definition. The first requirement of logical reasoning is that terms shall be accurately defined. There are, however, two ways in which the meaning of a term may be defined or explained. Every term, as we have already seen (§ 17), may be regarded either from the point of view of intension or from that of extension. To define in the usual sense is to explain from the standpoint of intension, to state the attributes or qualities connoted by the term. The process of explaining terms with reference to the

[1] *Essay Concerning Human Understanding*, Bk. III, Ch. XI.

objects, or classes of objects, for which they stand is known as Division. We may include, then, under the general term definition, (1) *Intensive definition, or definition in the ordinary sense*, and (2) *Extensive definition or division.*

§ 20. **Definition.** — To define a term is to state its connotation, or to enumerate the attributes it implies. Thus we define a parallelogram as a quadrilateral figure whose opposite sides are parallel. A distinction is often made between *verbal* and *real* definition. When we merely wish to explain the meaning in which we intend to employ some word, we have verbal definition. But when it is the purpose of our assertion to state the real nature or essential characteristics of some object, the proposition employed is said to constitute a real definition. This distinction, though not without importance, cannot be regarded as ultimate. For we never define a word or term for its own sake merely, but in order to understand the nature of the objects to which it refers. Indeed a mere word, apart from its uses or from the things for which it stands, has no interest for us. In defining a term, then, we are always attempting to explicate or explain, more or less directly, the nature of a thing, or our idea about a thing.

Nevertheless there is an advantage in distinguishing propositions whose *immediate* purpose is to expound the meaning of a word from those which assert something directly of an object. 'Monarchy consists in the authority of one man over others', may be regarded as a verbal definition, because the purpose of the proposition is simply to explain the meaning of the subject term. On the other hand 'iron

is malleable' is a *real* definition (though not a complete one), because it does not primarily refer to the signification of the word 'iron', but to the real object to which the name is applied.

In this connection it is interesting to notice that a proposition amounting to nothing more than a verbal definition is sometimes put forward as if it were an assertion containing some real knowledge. The solemn commonplaces in which ignorant persons delight are often of this character. 'A republic is a government by the people', 'a just man will do what is right', 'if it rains, the ground will be wet', may serve as examples. The mistake in such cases consists in supposing that these assertions are anything more than verbal. "Trifling propositions", is the name that Locke gives to this form of statement. "The property of water is to wet, and fire to burn; good pasture makes fat sheep, and a great cause of the night is the lack of the sun", are Corin's profound remarks to Touchstone, in summing up his philosophy.

There are two points of view from which the subject of definition may be considered. We may either discuss *the best method of obtaining real definitions of the nature of things*, or confine our attention to the *requirements which a good definition has to fulfil*. A person's ability to define either a term or the thing for which the term stands, depends, however, upon the possession of clear and distinct ideas on the subject. The problem, then, as to the best method of finding definitions, resolves itself into an inquiry concerning the means to be used in obtaining and systematizing our

ideas in general; and the answer to this question, so far as an answer can be given, must be found in the theory of logic as a whole. In this treatment of the subject we can consider only the requirements of a logical definition, and the rules which must be observed in stating it in language.

In Chapter II we briefly outlined the method proposed by Socrates and Plato for obtaining definitions. Consisting essentially of proceeding by means of question and answer, and compelling a speaker to admit particular facts which refute the general thesis he is maintaining, it came to be known by the name of Dialectic. By a consideration of individual cases, Socrates sought to obtain a definition which would be a complete and adequate expression of the nature of all the individuals sharing in the class name. Aristotle can therefore say, with some degree of truth, that it is to Socrates that we owe the method of induction and logical definitions. It should however be added that the Socratic use of induction as Plato represents it in his Dialogues is more often popular in character than strictly scientific, judged by our present standards.

The second question has reference to the formulation of a definition in language. Suppose that we already possess a clear conception of the meaning of the terms to be defined, what are the conditions that a logical definition must fulfil? The answer to this question is usually given in text-books of logic by means of a set of rules for definition. Before stating these rules, however, it is necessary to explain the meaning of certain terms which will be frequently employed

throughout the remainder of this chapter. These terms constitute what the older logicians called the Predicables, and state all the possible relations which a predicate may express with regard to a subject; *i.e.*, all the possible specifications of the general relationship of Predication.

According to the earliest classification, that of Aristotle, there are four such relations — Definition, Property, Genus or Differentia, and Accident. About 600 years after Aristotle, however, Porphyry revised this list, on the ground that Definition is not on a par with the others, since it is equivalent to the Genus plus the Differentia. For example, we have the definition, 'man is a rational (differentia) animal (genus)'. And man, the thing to be defined, Porphyry called the species.

Thus we have (1) the **Genus,** or any class containing two or more subordinate classes or species; (2) the **Species,** or the subordinate class of some wider whole; (3) the **Differentia,** or the qualities or characteristics distinguishing any term from other terms, from the genus to which it belongs, as well as from the species coördinate with it; (4) a **Property,** or any attribute not forming part of the connotation or definition of the term, but following from it, either as effect from cause or as conclusion from premise; (5) an **Accident,** or any attribute which is neither part of the connotation of a term nor necessarily connected with it. For example, 'plane figure bounded by straight lines' is the genus with reference to which square, rectangle, triangle, etc., are species. The differentia of a triangle is the characteristic of having three sides. In the proposition 'man is

capable of civilization' the predicate is a property, while the predicate in 'some men are lazy' is an accident, of the subject.

It is important to notice that the terms 'genus' and 'species' have not the same signification in logic as in the natural sciences. In classifying objects in natural history we use the terms 'variety', 'species', 'genus', 'family', and 'order', to denote varying degrees of relationship between certain groups or classes of objects. These terms, as thus employed, also indicate certain relatively fixed divisions, or permanent ways of grouping the various forms of plant and animal life. But in logic the terms 'genus' and 'species' are employed to indicate the relationship between any higher and lower class whatsoever. Moreover any term (excepting only the highest genus and the lowest species) may be regarded from different standpoints, as either a genus or a species. Thus 'man', for example, is a species of the genus 'animal'; but the same term also may be regarded as a genus including various species of men, Caucasians, Negroes, Mongolians, etc. In the same way 'animal' may be considered a species of the still more comprehensive class 'organized being', and this latter term again as a species of the genus 'material being'. A still higher or more comprehensive term which includes as its species material and spiritual beings alike is 'being'. Since this term includes everything that exists, and can therefore never be included in any more general class, it is sometimes called the highest genus (*summum genus*). On the other hand we might proceed downwards until we come to a

class not admitting of division into any subordinate classes. Such a term is called the lowest species (*infima species*).

We shall now proceed to state the requirements of a definition in terms of genus, species, and differentia: —

(1) *A definition should state the essential attributes of the thing to be defined.* This is done by stating the genus to which the object belongs, and also the peculiar marks or qualities by means of which it is distinguished from other members of the same class. Or as the rule is usually stated: A logical definition should give the next or proximate genus and the differentia of the species to be defined. Thus we define a triangle as a rectilinear figure (genus) having three sides (differentia); and man as an animal (genus) possessing the power of speech and reason (differentia).

(2) *A definition should not contain the name to be defined, nor any word directly synonymous with it.* If for example we were to define justice as the way of acting justly, or life as the sum of vital processes, we should be guilty of violating this rule.

(3) *The definition should be exactly equivalent to the class of objects defined; that is, it must be neither too broad nor too narrow.* In other words the definition must take account of the whole class and nothing but the class. 'A sensation is an elementary state of consciousness' is too broad a definition, since it applies equally to affective and connotative elementary processes. On the other hand the definition of government as 'an institution created by the people for the protection of their lives and liberties', is too narrow. It takes no account of absolute forms of government which do not depend upon

the will of the people. Each of these cases may also be regarded as a failure to give the true differentia of the class to be defined, and hence as violations of the first rule.

(4) *A definition should not be expressed in obscure, figurative, or ambiguous language.* The reasons for this rule are at once evident. Any lack of clearness or definiteness in a definition renders it useless as an explanation. Sometimes the words used in defining may be less familiar than the term to be explained (*ignotum per ignotius*). The definition once given of 'net' as 'a reticulated texture with large interstices or meshes', may serve as an example.

(5) *A definition should, whenever possible, be affirmative rather than negative.* A definition, that is, should state what a term implies rather than what it does not imply. Sometimes, however, the purpose of a definition may be best attained by a negative statement of what is excluded by the meaning of the term. Thus we may define a spiritual being as a being which is not material, that is, unlike a material body in that it is not made up of parts extended in space. This is an exception to the rule. But it should be noted that there are other definitions which, while negative in form, are not really exceptions to it. Such for instance is the definition of a bachelor as an unmarried man. This is a precise statement of what *is* included in the meaning of the term. It is therefore the meaning rather than the form of the definition to which we should look in applying this rule. The fault against which it is directed is that of the so-called 'infinite' definition, which merely states what a thing is not, without regard to whether such a negation sensibly increases one's

knowledge of the meaning of the term or not. Such a definition is 'infinite' in the sense that to enumerate everything that the term to be defined *is not* would be an infinite process.

The usual type of definition, as has been said, requires us to mention the proximate genus or next higher class to which the species to be defined belongs, and also the specific or characteristic differences that distinguish it from other species. Now it is clear that there are certain cases in which these conditions cannot be fulfilled. In the first place no such definition can be given of the highest genus, because there is no more general class to which it can be referred. And again, although it is possible to give the differentia of any species such as 'man' or 'metal', it is not possible to state *individual* characteristics by means of a definition. An individual thing may be perceived, and its various properties pointed out. But it is never possible to state in a formal definition wherein the individuality of a particular thing consists. The uniqueness of a particular object cannot be summed up in this way, but must be learned through perception. We may perhaps say that the highest genus is above, and the individual thing below, the sphere of ordinary definition. For these reasons, the *summum genus* and the *infima species* are sometimes said to be *indefinable.*

There are, moreover, other terms such as 'space', 'time', 'life', 'thought', which are not readily referred to any higher class, and for which, therefore, ordinary definitions cannot be given. These terms are sometimes said to denote objects that are *sui generis*, or of their own class.

Such, then, is the outcome of the traditional type of definition. But it should not be supposed that this is the only way in which good definitions can be reached. As a matter of fact the purposes and methods of the particular science or study employing the definition determine both its content and the proper form of its statement. The definition, by giving genus and specific differentia, is especially useful where our chief purpose is one of classification, of ranging the concepts employed in any subject in a fixed order for further reference and use. But from the point of view of our own times it suffers from various defects and deficiencies. It naturally belonged to that stage of our knowledge of the world which regarded it as composed of fixed species, or real and immutable kinds, so that classification represented the highest aim of scientific investigation. Only on the basis of this presupposition would it be possible to state once and for all what are the 'essential attributes' of the species to be defined. Modern science, on the other hand, finds it impossible to accept the existence of such an unchanging, hierarchical order of things, and tends to base its researches, instead, on the assumption that all things are undergoing some sort of evolutionary process of change. And in the second place, as we shall see more fully below, classification has given way to more profound methods and aims of knowledge. To say that 'man is a rational animal' no doubt enables us to grasp some of his relations to other animals, but comes far short of revealing his whole essential nature. And finally, as we have seen, there are on the traditional view certain terms which

cannot be defined at all — which have to be left on one side as 'indefinable'.

These considerations have led, in modern times, to the idea of what may be called a *systematic definition*. The aim of a systematic definition is to show the position of the object defined as an element in a system; that is, its relations both to other elements and to the system as a whole. Sometimes miscalled a genetic definition, its use is frequent where we are concerned with processes and the laws of their action, and it often represents an advance in knowledge upon classificatory definition. To define 'heat', for example, as 'a force in nature recognized in the phenomena of fusion and evaporation, etc.', tells us less about its real nature than the statement that it is 'a form of energy possessed by bodies *derived from* an irregular motion of their molecules'. To define 'water' as 'a fluid which descends from the clouds in rain', is less adequate for scientific purposes than the chemical definition of it as 'a fluid formed by adding one part of oxygen to two parts of hydrogen'. In zoölogy and botany the older definitions of animals and plants by genus and differentia received a new meaning in the light of the theory of evolution; for these classificatory relationships have been shown to be evidences and results of the degree of affinity in descent from common progenitors, and are revised accordingly. The definition of 'ape', for example, as a 'variety of the quadrumana having teeth like man, etc.', is widened to include less obvious characteristics; and this and other similarities to man, which the older definition merely stated, are now explained. In all such

cases the systematic definition tells us more about the real nature of the thing defined, because it relates the thing, through general laws of behavior, to other things and their characteristics. Again there are cases where either mode of definition seems equally adequate in itself, and we can employ either indifferently according to the purpose of the moment. In mathematics, for example, a circle may be defined equally well as 'a plane figure bounded by a line, all points of which are equally distant from a point within called the centre', or as 'the plane figure generated by revolving a straight line about one of its extremities which remains fixed'. And finally, we may mention a class of *genetic* definitions whose value seems merely practical, in that their purpose is only to give a brief statement of how to make a certain thing when it is wanted. Such are the chemical formulæ used in certain manufactures, or the recipes found in cook-books.

In addition to the question as to which mode of definition is to be preferred in any case, the further problem arises: What are the *essential* characteristics which the definition must state? This also must be determined by the purposes for which it is to be used. The essential characteristics of any subject will vary widely according to the different points of view from which it is examined. The legal definition of 'insanity', for example, differs from the medical. Jurisprudence is concerned here not with the study of mental abnormality as such, but with the determination of that degree of it which it is expedient to recognize as constituting irresponsibility for what would usually be con-

sidered as a criminal act, or as nullifying contracts, deeds, or wills. And in general we may say that the purpose of definitions in law is always to ensure that the original intention of the legislator shall be carried out, by stating as clearly as possible the distinguishing marks of the agents, acts, or states to which the law is intended to apply. This purpose, and not that of an exact statement of the nature of the thing defined, determines what shall be considered essential characteristics in its eyes. It is plain that there may often be, therefore, an important difference between a good legal definition and a good definition of the same subject-matter in another connection, for example. This example will also serve to illustrate the truth that it is neither necessary nor desirable that all definitions should be equally precise. A definition which from one point of view lacks logical completeness may sometimes be sufficiently exact for the purpose in hand. Such is the case with those definitions that are preliminary in any science or argument, and serve to outline its field and to prepare the way for further discussion. Too great haste in defining is almost as much a fault as failure to define at all; and there is a peculiar fallacy which attempts to bar the way to all fruitful discussion by remarking that 'it is all a question of definition, and if the terms had been first defined, all this argument would be unnecessary'. The remark is perfectly true, but it overlooks the fact that any fully adequate definition is the product of thinking, not its point of departure.

In sum, on this modern, more comprehensive and pro-

found view of definition, nothing is indefinable, for any object of thought belongs, as such, to some system or other, and can be both distinguished from and related to other objects within some systematic whole. Since classification is itself a simple, inadequate kind of systematization, the traditional form of definition in terms of genus, species and differentia now appears as a more elementary, partially successful attempt to realize the same ideal.

§ 21. **Division and Classification.** — We have already spoken of Division as a process of defining a term from the point of view of extension. This is to enumerate the objects or classes of objects which the term denotes. The enumeration must however be guided by certain principles which we have now to consider.

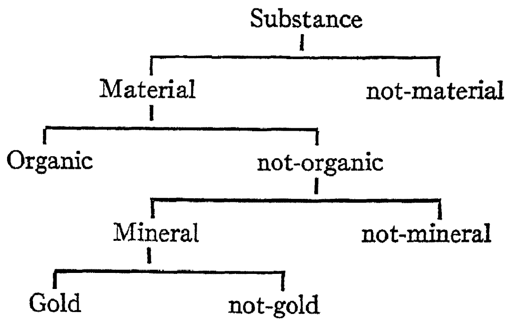
It is usual to begin by speaking of Dichotomy, or the division of a term into two parts (*δίχα τέμνειν*, to cut in two). This is a purely formal process, and is based on the so-called law of **Excluded Middle,** which is regarded as one of the fundamental laws of thought. The law may be stated as follows: There is no middle ground between contradictories. Any term, *a*, is either *b* or not-*b*. A triangle is either equilateral or not-equilateral. Of two contradictory predicates one or the other must belong to every possible subject.

Now it is clear that this is a purely formal principle of division. Some positive knowledge of the particular facts involved is always necessary in order to enable one to determine what things do stand in this relation of logical opposition. The logical law, in other words, does not help us at

all in deciding what may be regarded as not-*a* in any particular case. It is not, therefore, a means of increasing our knowledge, but merely a principle of order and arrangement.[1] This fact, obvious as it seems, was not understood by the Schoolmen who busied themselves with logic in the latter part of the Middle Ages. They clung firmly to the belief that it is possible to discover the nature of particular facts by purely formal operations of this kind. Accordingly they spent a great deal of time in classifying and arranging terms as contradictories, contraries, etc. Such work was doubtless of service in fixing the meaning of terms and in preventing confusion in their employment. But it was a purely verbal investigation, and could not lead to any discoveries regarding the nature of things.

Moreover it must be noticed that we do not always get propositions to which any meaning can be attached by uniting subjects and predicates in this way. If the law of dichotomy is not guided by knowledge of the particular facts, it will give absurd propositions like 'virtue is either square or not-square', 'iron is either pious or not-pious'. But unmeaning propositions being left out of account, we may proceed to divide everything according to this principle. All geometrical figures are either rectilinear or not-rectilinear; all rectilinear figures, either triangular or not-triangular; all triangles, equilateral or not-equilateral, etc. This method of division may be represented as follows: —

[1] See above, pp. 26, 27.



If it were desirable, the terms 'not-material', 'organic', and 'not-mineral' might also be further subdivided in the same way.

Now it is not difficult to see that the practical use of this principle will depend upon our ability to find some positive value for the negative not-*a*. That is, to make the law of more than formal value we must know what concrete term excludes *a*, or is its logical contradictory. And knowledge of this kind comes, as already said, only from experience of the particular facts. The strictly *logical* contradictory of *a* is always not-*a;* of wise, not-wise; of cold, not-cold; etc. Mistakes frequently arise in stating contradictories in a positive form. The difficulty is that terms are chosen which are not true logical contradictories. Thus if we say that every man is either wise or foolish, our terms are not contradictories, for a middle ground between them is possible. The same would be true of divisions like 'large or small', 'rich or poor', 'saint or sinner', 'idle or diligent'. In general it is well to scrutinize all dichotomic divisions very sharply, to see that the alternatives are really contradictories.

The method of dichotomy depends, as we have seen, upon the law of Excluded Middle. But there is also another process called **Division** in logic, which is perhaps better known by its less technical name of **Classification.** In Classification, which may be regarded as a more developed type of Division, there is no necessary limit to the number of classes or divisions that may be obtained. In this respect it differs fundamentally from the twofold division which we have been examining. Furthermore, a classification is always made according to some principle that is retained throughout the whole process. Any common characteristic of the group of individuals to be divided may be taken as a principle of classification. If however the characteristic chosen is merely an external and accidental one, the classification based upon it will be regarded as *artificial*, and made for some special or temporary purposes. Thus we might divide all flowering plants according to the color of the flowers, or the persons in any company according to the pattern of their shoes. A classification proceeding upon such surface distinctions obviously has no real or scientific value, except as it aids us to discover more fundamental or deep-lying resemblances between the individuals with which it deals, of which we may regard these superficial qualities as signs. Such a preliminary classification corresponds to what we might call a 'diagnostic' definition.

A scientific or natural classification, on the other hand, has for its purpose the statement of real likeness or resemblance. It seeks to find and group together the things related in some essential point. Consequently it selects as

CHAPTER VIII

THE CATEGORICAL SYLLOGISM

§ 32. **The Nature of Syllogistic Reasoning.** — The syllogism, as we have already seen, presents a conclusion together with the reasons by which it is supported. A single proposition taken by itself is dogmatic: it merely asserts, without stating the grounds upon which it rests. A syllogism, on the other hand, justifies its conclusion by showing the premises from which it has been derived. It thus appeals to reason and compels assent. But of course the premises of a syllogism must be taken for granted. If they are disputed or doubtful, the argument is pushed a step further back, and it is first necessary to show the grounds upon which they rest. The assumption of syllogistic reasoning — and, indeed, of all reasoning whatsoever — is that it is possible to reach propositions that every one will accept. There are certain facts, we say, well known and established, and these can always be appealed to in support of our conclusions. In syllogistic reasoning, then, we exhibit the interdependence of propositions; *i.e.*, we show how the truth of some new proposition, or some proposition not regarded as beyond question, follows necessarily from other propositions whose truth every one will admit.

Hence one question connected with the syllogism is this: Under what conditions do propositions accepted as

true contain or imply a new proposition as a conclusion? Or we may put the question in this form: In what ways may the four kinds of logical propositions, A, E, I, O, be combined so as to yield valid conclusions?

We pointed out in a previous chapter that a syllogism has always two premises. It is impossible however to obtain a conclusion by combining any two propositions at random, as *e.g.*, —

All A is B,
No X is Y.

It is evident that *any* two propositions will not yield a conclusion by being taken together. In order to serve as premises for a categorical syllogism propositions must fulfil certain conditions, and stand in certain definite relations to each other. To determine some of the most apparent of these conditions, let us examine the argument: —

All mammals are vertebrates,
The whale is a mammal,
Therefore the whale is a vertebrate.

It will be noticed that the term 'mammal' is common to both premises, and that it does not occur at all in the conclusion. Moreover, it is because the other terms are compared in turn with this common or **Middle Term** and found to agree with it, that they can be united in the conclusion. It is only propositions having a middle term, therefore, which can be employed as premises. The categorical syllogism is thus essentially a process of comparison. Each of the terms entering into the conclusion is compared in turn

with the same middle term, and in this way their relation to each other is determined. We reach the conclusion not directly or immediately, but by means of the middle term. The conclusion is therefore said to be *mediated*, and the process itself is sometimes called **Mediate Reasoning.**

It will be interesting to compare what has just been said regarding the function of the middle term with what has been previously stated regarding the nature of inference. When we infer one fact from another, it was said, we do so by discovering some identical link or connecting thread which unites both. We may say that to infer is to see that, in virtue of some identical link which our thought has brought to light, the two facts, or groups of facts, are in a certain sense identical. Now the middle term in a categorical syllogism is just the explicit statement of the nature of this identical link. It is true that in such a syllogism we seem to be operating with words or terms rather than with the thought-process itself. When we go behind the external connection of the terms, however, we can see that the middle term represents the universal principle by means of which the conclusion is reached. In the example given above, for instance, we reason that the whale, *being a mammal*, is a vertebrate.

The terms that enter into the conclusion of a syllogism are sometimes called the *Extremes*, as opposed to the middle term. Of the Extremes, *the predicate of the conclusion is known as the* **Major Term,** *and the subject of the conclusion as the* **Minor Term.** The premise containing the major term is called the **Major Premise,** and stands first when

the syllogism is arranged in logical order. The **Minor Premise,** containing the minor term, stands second. In actual reasoning, however, the propositions of which the syllogism is composed may occur in any order; either premise, or even the conclusion, may stand first. To arrange an argument, therefore, it is necessary to determine which is the major, and which is the minor premise. This can be done most readily by turning to the conclusion and distinguishing the major and minor terms. For example, take the syllogism: —

> The whale suckles its young,
> No fish suckles its young,
> Therefore the whale is not a fish.

By turning to the conclusion we see that 'fish' (being the broader term and therefore naturally predicate) is the major term. The proposition containing this term, 'no fish suckles its young', is therefore the major premise, and should stand first. Before proceeding to examine the syllogism further it would be desirable to arrange it as follows: —

> No fish is an animal which suckles its young,
> The whale is an animal which suckles its young,
> Therefore the whale is not a fish.

§ 33. **The Rules of the Syllogism.** — It is customary to give a number of rules or canons to which the categorical syllogism must conform in order to yield valid conclusions. We shall first enumerate the rules and afterwards remark on their meaning and importance.

morphic rocks are sedimentary rocks'. This may be clearly shown by representing the propositions by Euler's method of circles as in Fig. 6. We know from the second proposition that the circle representing 'metamorphic rocks' falls partly within the circle of 'stratified substances'. But it is impossible to determine from the statement whether it corresponds at all with the circle of sedimentary rocks, or falls, as in the figure, entirely without it.

The fourth rule states that no term may be distributed in the conclusion which was not distributed in one of the premises. That is, the conclusion must be proved *by means of* the premises, and accordingly no term which was not employed in its universal signification in the premises may be used universally or distributively in the conclusion. This rule may be violated by using either the major or the minor term in a wider sense in the conclusion than in the premise in which it occurs. The resulting fallacies are then known as the **Illicit Process** of the major and minor terms respectively. As an illustration of the illicit process of the major term we may consider the following argument: —

> All rational beings are responsible for their actions,
> Brutes are not rational beings,
>
> ---
>
> Therefore brutes are not responsible for their actions.

It will be at once seen that the major term, 'beings responsible for their actions', is distributed in the conclusion, but was not distributed when it appeared as the predicate of an affirmative proposition in the major premise. The fallacious

nature of this argument may also be shown by means of circles.

The illicit process of the minor term is usually more easily detected. We may take as an example: —

All good citizens are ready to defend their country,
All good citizens are persons who vote regularly at elections,

---

Therefore all who vote regularly at elections are ready to defend their country.

It is clear that the minor term, 'persons who vote regularly at elections', is undistributed when used as the predicate of the minor premise. In the conclusion, however, it is wrongly taken universally, and it is this unwarranted extension to which the name of **Illicit Minor** is given. Here again Euler's circles will clearly reveal the nature of the mistake.

The fifth and sixth rules have reference to negative premises. It is not difficult to understand why two negative premises cannot yield any conclusion. For, from the fact that S and P are both excluded from M, we can conclude nothing regarding their relation to each other. Two negative premises afford no standard by means of which we can determine anything concerning the relation of the major and minor terms. Again, where one premise is negative and the other affirmative, it is asserted that of the major and minor terms, one agrees, and the other does not agree, with the middle term. The necessary inference from these premises is that the major and minor terms do not agree with each other. That is, the conclusion must be negative.

It is worth noticing that it is sometimes possible to obtain a conclusion from premises which are both negative in form. For example: —

> No one who is not thoroughly upright is to be trusted,
> This man is not thoroughly upright,
>
> ---
>
> Therefore this man is not to be trusted.

In this example, although the form of both premises is negative, the minor premise supplies a positive basis for argument, and is really affirmative in character. Or we may say that the 'not' in the predicate of the minor premise belongs to the predicate, and not to the copula. The proposition may therefore be said to affirm rather than to deny.

The seventh and eighth rules can be proved by considering separately all the possible combinations of particular premises. If this is done it will be found that these rules are direct corollaries from the third and fourth, which are concerned with the proper distribution of terms. It is impossible to secure the necessary distribution with two particular premises; for either the distribution of the middle term will not be provided for, or if this has been secured by means of a negative premise, the argument will involve an illicit process of the major term. By means of the same rules it may be proved that a particular premise always requires a particular conclusion. The truth of these two subordinate canons also may be readily shown by circles.

§ 34. **The Figures of the Syllogism.** — We have seen what an important part the middle term plays in the cate-

gorical syllogism. It constitutes the mediating link between the major and minor terms, and makes possible their union. Now upon the position of the middle term in the premises depends the **Figure** of the syllogism. There are four possible arrangements of the middle term in the two premises, and therefore four figures. If we let P represent the major term, S the minor, and M the middle term, the different figures may be represented as follows: —

FIRST FIGURE	SECOND FIGURE
M — P	P — M
S — M	S — M
∴ S — P	∴ S — P

THIRD FIGURE	FOURTH FIGURE
M — P	P — M
M — S	M — S
∴ S — P	∴ S — P

In the first figure the middle term is the subject of the major premise and the predicate of the minor premise.

In the second figure the middle term is predicate of both major and minor premises.

The third figure has the middle term as the subject of both premises.

In the fourth figure the middle term occupies just the opposite position in the two premises to that which it holds in the first figure; *i.e.*, it is the predicate of the major premise and the subject of the minor premise.

EXERCISES (VIII)

1. (*a*) What is the function of the categorical syllogism? (*b*) Explain and illustrate Aristotle's statement to the effect that science is a search for the middle term.

2. How do you distinguish the Minor, Major, and Middle Terms of a syllogism?

3. Illustrate the fallacies mentioned in the text (*a*) by means of circles; (*b*) by means of arguments.

4. Prove the seventh and eighth rules of the syllogism.

5. Arrange the following arguments in the regular logical order of major premise, minor premise, and conclusion, and examine them to see whether they conform to the canons of the syllogism. If they fail to conform, tell what fallacies result.

(1) Gold is not a compound substance; for it is a metal, and none of the metals are compounds.

(2) All cruel men are cowards, no college men are cruel, therefore no college men are cowards.

(3) This man shares his money with the poor, but no thief ever does this, therefore this man is not a thief.

(4) Some useful metals are becoming rarer. Iron is a useful metal, and is therefore becoming rarer.

(5) He who is content with what he has is truly rich. An envious man is not content with what he has; no envious man therefore is truly rich.

(6) Some F are not H, no F are G, no G are H.

CHAPTER IX

THE VALID MOODS AND THE REDUCTION OF FIGURES

§ 35. **The Moods of the Syllogism.** — By the **Mood** of a syllogism we mean the combination of propositions A, E, I, and O, which goes to make it up. When a syllogism consists of three universal affirmative propositions its mood is AAA; if it is composed of a universal negative, a particular affirmative, and a particular negative proposition, its mood is EIO.

Every syllogism, as has been already stated, involves some arrangement of the four propositions, A, E, I, O, taken three at a time. Now there are in all sixty-four possible permutations of these four propositions when so taken. We might then write out these sixty-four moods, and proceed to determine which of them are valid. But this would be a long and somewhat tedious undertaking. Moreover, if we can determine which are the valid combinations of premises, we can draw the proper conclusions for ourselves. Since there are but two premises in each syllogism, we shall have to deal only with the possible permutations of A, E, I, and O, taken two at a time, or with sixteen combinations in all.

The following, then, are the only possible ways in which the propositions A, E, I, and O can be arranged as premises: —

AA	EA	IA	OA
AE	~~EE~~	IE	~~OE~~
AI	EI	~~II~~	~~OI~~
AO	~~EO~~	IO	~~OO~~

Some of these premises, however, cannot yield conclusions, since they plainly violate certain rules of the syllogism. The combinations of negative premises, EE, EO, OE, and OO, can be at once struck out. And since no conclusion follows from two particular premises we can eliminate II, IO, and OI. There remain for further consideration the combinations: —

AA	EA	IA	OA
AE	—	IE	—
AI	EI	—	—
AO	—	—	—

At this point, recalling the fact that every argument must belong to one of the four figures, we are confronted by the question: Which of the above combinations of premises will yield valid conclusions in the first, second, third, and fourth figures, respectively? By examining the form of the syllogism in each of these figures we shall be able to discover what conditions must be fulfilled in each case, and to lay down special canons for each figure. We shall first proceed to state and prove the special canons of the different figures. It is useful to commit these rules to memory, although the student can, of course, always derive them for himself when time permits.

§ 36. **The Special Canons of the Four Figures.** — *In the first figure the minor premise must be affirmative, and the major premise universal.*

The first figure is of the form: —

$$\begin{array}{l} \quad M - P \\ \quad S - M \\ \hline \therefore S - P \end{array}$$

To show that the minor premise is affirmative we employ the indirect method of proof. Let us suppose that the minor premise is not affirmative, but negative. Then since one premise is negative, the conclusion must be negative. But if the conclusion is a negative proposition, its predicate, P, must be distributed. Any term distributed in the conclusion must, however, have been distributed when it was used in the premise. P must be distributed, therefore, as the predicate of the major premise. But since negative propositions alone distribute their predicates, the major premise, M — P, must be negative. But by hypothesis the minor premise, S — M, is negative. We therefore have two negative premises, which is impossible. Our supposition that the minor premise is negative is therefore false; or in other words the minor premise must be affirmative.

This having been established, we can very easily prove that the major premise must be universal. For the middle term, M, must be distributed in at least one of the premises. But it is not distributed in the minor premise, for it is there the predicate of an affirmative proposition. It must therefore be distributed as the subject of the major premise, that is, the major premise must be universal.

If we turn now to the second figure, we shall find that the

following rules may be deduced from a consideration of its form: —

(1) *One premise must be negative, and the conclusion therefore negative.*

(2) *The major premise must be universal.*

The second figure is in the form: —

$$\begin{array}{r} \text{P} — \text{M} \\ \text{S} — \text{M} \\ \hline \therefore \text{S} — \text{P} \end{array}$$

The reason for the first rule is at once evident. If one premise is not negative, the middle term, M, is not distributed, and no conclusion is possible. The only means of securing distribution of the middle term in the second figure is by means of a negative premise. And if one premise is negative, it follows that the conclusion must be negative.

This having been established, the proof of rule 2 follows almost immediately. For since the conclusion is negative its predicate, P, must be distributed. And since P is distributed in the conclusion, it must have been used distributively when it occurred as the subject of the major premise, or, in other words, the major premise must be universal..

The third figure is of the form: —

$$\begin{array}{r} \text{M} — \text{P} \\ \text{M} — \text{S} \\ \hline \therefore \text{S} — \text{P} \end{array}$$

From an analysis of this, the two following rules may be obtained: —

(1) *The minor premise must be affirmative.*

(2) *The conclusion must be particular.*

The minor premise is here shown to be affirmative by the method employed in proving the same rule in the first figure. That is, we suppose the minor premise negative and show that the conclusion is therefore negative, and the major term distributed. It follows that the major term must be distributed as the predicate of the major premise. But this could happen only if the major premise were negative. The hypothesis that the minor premise is negative thus leads to the absurdity of two negative premises. The fact that the opposite is true, that the minor premise is affirmative, is therefore proved indirectly.

Since the minor premise is affirmative, its predicate S is undistributed. S must therefore be used in an undistributed, *i.e.*, particular sense in the conclusion. And, as this term forms its subject, the conclusion is particular.

In the fourth figure the terms are arranged in the following way: —

$$\begin{array}{c} P - M \\ M - S \\ \hline \therefore S - P \end{array}$$

From the form of this figure we can derive the following special canons: —

(1) *If either premise is negative, the major premise must be universal.*

(2) *If the major premise is affirmative, the minor must be universal.*

(3) *If the minor premise is affirmative, the conclusion must be particular.*

The student will be able to prove these canons for himself by applying the rules of the syllogism in the same way as has been done in the proofs already given.

§ 37. **The Determination of the Valid Moods in Each of the Figures.** — We have now to apply these special canons in order to determine what moods are valid in each of the four figures. It has already been shown (p. 147) that the premises which are not excluded by the general rules of the syllogism are: —

AA	EA	IA	OA
AE	—	IE	—
AI	EI	—	—
AO	—	—	—

Now we have proved that in the first figure the major premise must be universal, and the minor affirmative. The only combinations of premises which will stand these tests are, AA, EA, AI, and EI. Drawing the proper conclusion in each case, we have as the four valid moods of the first figure: —

AAA, EAE, AII, EIO.

It will be noticed that the first figure enables us to obtain as conclusion any one of the four logical propositions A, E, I, and O.

The special canons of the second figure state that the major premise must be universal, and one premise negative.

Selecting the combinations of premises that fulfil these conditions, we obtain EA, AE, EI, and AO. These give, when the conclusions have been drawn, the following four moods of the second figure: —

EAE, AEE, EIO, AOO.

By means of the second figure, therefore, we are able to establish the truth only of the negative propositions, E and O.

In the third figure the minor premise must be affirmative, and the conclusion particular. Taking all the combinations in which the minor is affirmative, we have AA, IA, AI, EA, OA, EI. It must be remembered that the third figure yields only particular conclusions, even where both premises are universal. The valid moods in this figure are therefore as follows: —

AAI, IAI, AII, EAO, OAO, EIO.

Those canons of the fourth figure having to do with the premises state that where either premise is negative a universal major is necessary, and that an affirmative major premise must be accompanied by a universal minor. The combinations of propositions which fulfil these conditions are AA, AE, IA, EA, and EI. In drawing conclusions from these premises, however, it is necessary to pay attention to the third canon of this figure, which states that where the minor premise is affirmative, the conclusion must be particular. Accordingly, the valid moods of this figure may now be written: —

AAI, AEE, IAI, EAO, EIO.

Here we are able to obtain a universal negative as a conclusion, but not a universal affirmative. It is interesting to notice that the first figure alone enables us to prove a proposition of the form A.

It may also be pointed out that the combination IE, although not excluded by the general rules of the syllogism, cannot be used at all as a premise, since it violates the canons of all four figures. There remain in all, then, nineteen valid moods of the syllogism — four in the first figure, four in the second, six in the third, and five in the fourth figure.

§ 38. **The Mnemonic Lines.** — It is not necessary to commit to memory the valid moods in each figure. By applying the general rules of the syllogism to the figure in question, the student will be able to determine for himself in every case whether or not an argument is valid. The Latin Schoolmen in the thirteenth century, however, invented a system of curious mnemonic verses for the purpose of making it easy to remember the valid moods in each figure. Although it is not necessary for the student to burden his memory with these barbarous names, it is interesting to understand the use of the lines: —

> *Barbara*, *Celarent*, *Darii*, *Ferio*que prioris;
> *Cesare*, *Camestres*, *Festino*, *Baroko*, secundæ;
> Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
> *Bokardo*, *Ferison*, habet; Quarta insuper addit
> *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

The words printed in ordinary type are real Latin words, indicating that the four moods represented by Barbara, Celarent, Darii, and Ferio are the valid moods of the first

figure, that the next four are valid in the second figure, that the third figure has six valid moods represented by as many artificial names, and that the fourth figure adds five more. Each word represents a mood, the vowels A, E, I, and O indicating the quality and quantity of the propositions which go to compose it. Thus Barbara signifies the mood of the first figure which is made up of three universal affirmative propositions AAA; Cesare, a mood of the second figure, composed of the three propositions EAE. These lines, then, sum up the results reached on pages 151–153 regarding the valid moods in each figure.

But further, certain consonants in these mnemonic words indicate how arguments in the second, third, or fourth figures may be changed to the form of the first figure. The first figure was called by Aristotle the **Perfect Figure,** and the second and third the **Imperfect Figures,** since he did not regard an argument in these forms as so direct and convincing as one of the first-mentioned type. The fourth figure was not recognized by Aristotle, but is said to have been introduced into logic by Galen, the celebrated teacher of medicine, who lived in the latter half of the second century. If we consider an example of this figure, the reason for refusing it an equal rank with the other three will appear: —

> The whale is a mammal,
> All mammals are vertebrates,
> Therefore some vertebrates are whales.

It is plain that the conclusion of the argument is somewhat strained. That is, it would be more natural to obtain

the conclusion 'whales are vertebrates', than to infer that 'some vertebrates are whales'; for the latter statement seems to make the species, or less inclusive term, the predicate of the genus, or wider term. It would therefore appear that the reason why Aristotle omitted the fourth figure was that it improperly makes the real major term a minor, and the real minor a major, and so states in a less adequate way an argument which could be expressed more effectively in figure one.

The process of changing an argument from one of the so-called imperfect figures to the first figure is known as **Reduction.** And as we have said, the mnemonic lines give rules for carrying out this process. For example, *s* indicates that the proposition represented by the preceding vowel is to be converted simply. Thus an argument of the mood Cesare in the second figure is changed to Celarent in the first figure by converting the major premise simply. Again, *p* denotes that the preceding proposition is to be converted by limitation, or *per accidens; m* is supposed to stand for *mutare*, and indicates that the premises are to be transposed; *k*, which is used in the moods Baroko and Bokardo, shows that an indirect method of reduction is necessary to change the arguments to the first figure.

Further, the initial consonants of the moods of the imperfect figures correspond with those of the moods in the first figure, to which they can be reduced. Cesare and Camestres of the second figure, for example, and Camenes of the fourth are reducible to Celarent; and, similarly, Festino, Felapton, Fesapo, and Fresison may all be reduced to Ferio.

The student who understands the structure of the syllogism will be able to arrange an argument in one figure or another, as may be most convenient, without the aid of any mechanical rules. It may be interesting, however, to give a single example for the sake of illustrating the workings of this most ingenious device. Let us take the following argument in Camestres: —

All members of the class are prepared for the examination,
No idle persons are prepared for the examination,

Therefore no idle persons are members of the class.

Now the *m* in Camestres shows that the major and minor premises are to be transposed; the first *s* indicates that the minor premise is to be converted, and the second that the same process must be performed on the conclusion.

Converting the minor premise and conclusion and transposing the premises, we obtain: —

No persons prepared for the examination are idle,
All members of the class are prepared for the examination
Therefore no members of the class are idle persons.

The result, as will at once be seen, is an argument in Celarent, of the first figure.

EXERCISES (IX)

1. With respect to what aspects of its contained propositions are the rules of the syllogism determined?

2. What is the relation of the Moods to the Figures of the syllogism?

3. What two methods may be adopted to determine whether a particular argument is valid in a given figure?

4. Prove the special canons of the fourth figure.

5. Name the premises from which valid conclusions may be drawn, no account being taken of the figures: —

AA, EO, IA, IO, II, EE, EI, AE, EA, OO.

6. 'The middle term must be distributed once at least'. In what figures may it be distributed twice? What is the character of the conclusion when this occurs?

7. (*a*) Prove generally that when the major term is predicate in its premise, the minor premise must be affirmative. (*b*) If the major term be distributed in its premise, but used undistributively in the conclusion, determine the mood and figure.

8. Explain why we can obtain only negative conclusions by means of the second figure and particular conclusions by means of the third.

9. What conclusions do AA, AE, and EA yield in the fourth figure? Explain.

10. Is it possible for both major and minor terms to be undistributed at the same time in the premises? If so, construct an argument where this is the case.

11. If the major premise and the conclusion of a valid syllogism agree in quantity but differ in quality, determine by general reasoning the mood and figure.

12. What is Reduction? Reduce the following argument to the first figure: —

Almost all criminals are mentally diseased,
All criminals are subject to punishment,
Therefore some persons subject to punishment are mentally diseased.

CHAPTER X

HYPOTHETICAL AND DISJUNCTIVE ARGUMENTS

§ 39. **The Hypothetical Syllogism.** — We have hitherto been dealing with syllogisms composed entirely of categorical propositions, and have not referred to the use which is made of conditional propositions in reasoning. A conditional proposition is sometimes (not altogether happily) defined as the union of two categorical propositions by means of a conjunction. It is the expression of an act of judgment which does not directly or unambiguously assert something of reality. We have already pointed out that there are two classes of conditional propositions: the *hypothetical* and the *disjunctive*, and corresponding to these we have the **Hypothetical** and the **Disjunctive Syllogism.** The hypothetical syllogism has a hypothetical proposition as a major premise and a categorical proposition as a minor premise. The disjunctive syllogism in the same way is composed of a disjunctive proposition as major, and a categorical proposition as minor, premise. In addition to these, we shall have to treat of another form of argument called the 'dilemma', made up of hypothetical and disjunctive propositions.

A hypothetical proposition does not assert directly the existence of a fact, but states the connection between a supposition or condition and its consequence. It is usually introduced by some word or conjunctive phrase like 'if', 'sup-

posing', or 'granted that'; as, *e.g.*, 'if he were to be trusted, we might give him the message'; 'suppose that A is B, then C is D'. The part of a hypothetical proposition which expresses the supposition or condition is known as the **Antecedent**; the clause stating the result is called the **Consequent.** Thus in the proposition 'he would write if he were well', the consequent, 'he would write', is stated first, and the antecedent, 'if he were well', follows.

The hypothetical syllogism, as has been already remarked, has a hypothetical proposition as its major, and a categorical proposition as its minor, premise: —

If justice is to prevail, his innocence will be proved,
And justice will prevail,

Therefore his innocence will be proved.

It will be noticed that in this argument the minor premise *affirms the antecedent*, and that as a result the conclusion affirms the consequent. This form is known as the *constructive hypothetical syllogism*, or the *modus ponens*.

In the following example it will be observed that the *consequent is denied*, and the conclusion obtained is therefore negative.

If he were well, he would write,
He has not written,

Therefore he is not well.

This is called the *destructive hypothetical syllogism*, or *modus tollens*.

The rule of the hypothetical syllogism may therefore be stated as follows: *Either affirm the antecedent or deny the*

consequent. If we affirm the antecedent, *i.e.*, declare that the condition exists, the consequent necessarily follows. And on the other hand if the consequent is declared to be non-existent we are justified in denying that the condition is operative.

The violation of these rules gives rise to the fallacies of *denying the antecedent*, and of *affirming the consequent.* Thus we might argue: —

> If he were well, he would write,
> But he is not well,
>
> ---
>
> Therefore he will not write.

Here the antecedent is denied and the argument plainly false. For we cannot infer that his being well is the only condition under which he would write. We do not know, in other words, that the antecedent stated here is the only, or essential condition of the consequent. We know that if there is fire there must be heat; but we cannot infer that there is no heat when no fire is present. Of course if we can be certain that our antecedent expresses the essential condition, or real *sine qua non* of the consequent, we can go from the denial of the former to that of the latter. For example: —

> If a triangle is equilateral, it is also equiangular,
> This triangle is not equilateral,
>
> ---
>
> Therefore it is not equiangular.

To illustrate the fallacy of affirming the consequent we may take the following example: —

If perfect justice prevailed, the rich would not be permitted to rob the poor,

But the rich are not permitted to rob the poor,

Therefore perfect justice prevails.

Here the antecedent states only *one* condition under which the consequent may follow. Because the consequent is declared to exist, it is by no means necessary that it should exist *as a consequence* of the operation of this condition. It is also worth noting in this example that the consequent of the major premise is negative. The minor premise *which affirms the consequent* also takes a negative form. To deny the consequent we should have to say, 'the rich are permitted to rob the poor'. Or to put the matter generally, it is necessary to remember that the affirmation of a negative proposition is expressed by a negative proposition, and that the denial of a negative — the negation of a negation — is of course positive in form.

§ 40. **Relation of Categorical and Hypothetical Arguments.** — It is evident that the form of the hypothetical syllogism is very different from that of the categorical. But although this is the case, it must not be supposed that with the former we have passed to a wholly distinct type of reasoning. In hypothetical reasoning, as in categorical, it is the presence of a universal principle which enables us to bring into relation two facts which formerly stood apart. Indeed it is often a matter of indifference whether the argument is stated in one form or the other. Thus we may argue in hypothetical form: —

If a man is industrious, he will be successful,
A is an industrious man,

Therefore A will be successful.

The same argument may however be expressed equally well in categorical form: —

All industrious men will be successful,
A is an industrious man,

Therefore A will be successful.

It is clear that whatever the form in which the argument is expressed, the reasoning remains essentially the same. The middle term, or general principle that makes it possible to unite the subject and predicate of the conclusion, in the hypothetical as well as in the categorical syllogism, is 'industrious'. A will be successful, we argue, because he is industrious, and it is a rule that industrious men are successful.

Moreover, if an argument is fallacious in one form, it will also be fallacious when expressed in the other. The defects of an argument cannot be cured simply by a change in its form. When an hypothetical argument in which the antecedent is denied is expressed categorically, we have the fallacy of the illicit major term. Thus, to state the example of denying the antecedent given on page 160, we get: —

The case of his being well is a case of his writing,
The present is not a case of his being well,

Therefore the present is not a case of his writing.

Similarly when an argument in which the consequent is affirmed is changed to the categorical form the defect in the reasoning appears as the fallacy of undistributed middle: —

> If this tree is an oak, it will have rough bark and acorns,
> This tree has rough bark and acorns,
> ______________________________
> Therefore it is an oak.

When this argument is expressed in categorical form it is at once clear that the middle term is not distributed in either the major or minor premise: —

> All oak trees are trees having rough bark and acorns,
> This tree is a tree having rough bark and acorns,
> ______________________________
> Therefore this tree is an oak.

The change from the categorical to the hypothetical form of argument, then, does not imply any essential change in the nature of the reasoning process itself. Nevertheless it is important to note that hypothetical propositions and hypothetical arguments emphasize an aspect of thinking which is inadequately expressed by the categorical syllogism. When dealing with the extension of terms we pointed out that every term, as actually used in a proposition, has both an extensive and an intensive function. That is, the terms of a proposition are employed both to name certain objects or groups of objects, and to connote or imply certain attributes or qualities. In the proposition 'these are oak trees', the *main* purpose is to identify the trees given in perception with the class of oak trees. When on

the other hand we say 'ignorant people are superstitious', the proposition does not refer directly to any particular individuals, but states the necessary connection between ignorance and superstition. Although the existence of ignorant persons who are also superstitious is *presupposed* in the proposition, its most prominent function is to assert a connection of attributes which is wholly impersonal. We may perhaps say that in spite of the categorical form the proposition is essentially hypothetical in character. Its meaning might very well be expressed by the statement, 'if a man is ignorant, he is also superstitious'. What is here emphasized is not the fact that ignorant persons exist and are included in the class of superstitious persons, but rather the general law of the necessary connection of ignorance and superstition. The existence of individuals to whom the law applies is of course presupposed by the proposition. It is not however its main purpose directly to affirm their existence.

Thus we have reached the following position: Every judgment has two sides, or operates in two ways. On the one hand it asserts the existence of individual things and sets forth their qualities and relations to other things. But at the same time every judgment seeks to go beyond the particular case, and to read off a general law of the connection of attributes or qualities which shall be true universally. In singular and particular propositions the categorical element — the direct assertion of the existence of particular objects — is most prominent, although even here the hint or suggestion of a general law is not altogether

absent. When we reach the true universal proposition, however, the reference to particular things is much less direct, and the meaning seems capable of more adequate expression in hypothetical form.

Now in the chapters on the categorical syllogism this latter aspect of judgments was left out of account. Propositions were there interpreted as referring directly to objects, or classes of objects (cf. § 25). The proposition S is P, for example, was taken to affirm that some definite object, or class of objects, S, falls within the class P. And the fact that it is possible to apply this theory shows that it represents one side of the truth. But the student must sometimes have felt that in this procedure the most important signification of the proposition is lost sight of. It seems absurd to say, for example, that in the proposition 'all material bodies gravitate', the class of 'material bodies' is included in the wider class of 'things that gravitate'. The main purpose of the judgment is evidently to affirm the necessary connection of the attributes of materiality and gravitation. The judgment does not refer directly to things or classes of things at all, but asserts without immediate reference to any particular object, *if* material, *then* gravitating. The propositions of geometry are still more obviously hypothetical in character. 'The three angles of a triangle are equal to two right angles', for example, cannot, without violence, be made to mean that the subject is included in the class of things which are equal to two right angles. The main purpose of the proposition is obviously to assert the necessary connection of the 'triangu-

larity' and the equality of angles with two right angles, and not to make any direct assertion regarding any actually existing object or group of objects.

In sum, our thought is at once both categorical and hypothetical. As categorical, it refers directly to objects and their relations. The terms of the proposition are then taken in extension to represent objects or groups of objects, and the copula to assert the inclusion of the subject in the predicate, or, in cases of negative propositions, to deny this relation. As hypothetical, the reference to things is much more indirect. The terms of the proposition are no longer regarded as representing objects or classes, but are interpreted primarily from the point of view of *intension*. The judgment affirms or denies the connection of the qualities or attributes *connoted* by the terms, rather than that of the objects which they denote. Sometimes the one aspect of thought, sometimes the other, is the more prominent.

In sense-perception and in simple historical narration, assertions are made directly and categorically regarding things and events. The main interest is in particular objects, persons, or events, and our judgments refer directly and unambiguously to them. But as we have already seen, our thought from its very beginning attempts to get beyond the existence of particular things and events, and to discover what qualities of objects are necessarily connected. We pass from perception and observation to explanation, from the narration of events, to the discovery of the law of their systematic organization. And as a result of this advance our

judgments deal no longer exclusively with particular objects and events and the fact of their relation, but with the general laws of the connection between attributes and qualities. There is of course no fixed point at which we pass from the categorical to the hypothetical aspect of thinking. But in general, as we pass from judgments of sense-perception and memory to a statement of theories and laws, the hypothetical element comes more and more clearly into the foreground. We have seen that it is almost impossible to interpret propositions regarding geometrical relations as referring directly to classes of objects. In the same way it is evident that propositions stating general laws are more truly hypothetical than categorical. When we assert that 'all men are mortal' the proposition does not intend to state a fact in regard to each and every man, or to refer directly to individuals at all, but to express the essential and necessary relation between humanity and mortality. A proposition essentially hypothetical in character may of course be expressed in categorical form. It must be remembered, however, that it is not the form, but the purpose or function of a proposition, which determines its character. Nevertheless the hypothetical form does justice to an aspect of thought which is especially prominent in the universal laws and formulas of scientific knowledge, and which is not adequately represented by the theory of subsumption, or the inclusion of the subject in the predicate.

§ 41. **Disjunctive Arguments.** — A disjunctive proposition, as we have already seen, is of the form, 'A is either B, or C, or D'; a triangle is either right-angled, obtuse-

angled, or acute-angled. It is sometimes said to be the union of a categorical and a hypothetical proposition. On the one hand it asserts categorically regarding A, and without reference to any external condition. But the disjunctive proposition is not simple like the categorical proposition: it states its results as a series of systematically related conditions and consequences. If A is not B, it tells us, it must be either C or D; and if it is C, it follows that it cannot be B or D.

A disjunctive proposition may at first sight appear to be a mere statement of ignorance, and, as such, to be less useful than the simple categorical judgment of perception. And it is true that the disjunctive form may be employed to express lack of knowledge. 'I do not know whether this tree is an oak or an ash'; 'he will come on Monday or some other day'. A true disjunctive proposition, however, is not a mere statement of ignorance regarding the presence or absence of some fact of perception. It is an attempt on the part of intelligence to determine the whole series of circumstances or conditions within which any fact of perception may fall, and to state the conditions in such a way that their systematic relations are at once evident. And to do this positive knowledge is necessary. In the first place the enumeration of possibilities must be exhaustive, no cases must be overlooked, and no circumstances left out of account. Secondly the members of the proposition must be taken so as to be really disjunctive. That is, they must be exclusive of one another. We cannot combine disjunctively any terms we please, as 'perhaps this' *or* 'perhaps that'.

But it is only when we understand the systematic connections of things in the field in question that we are able to express these connections in the form, *either* B *or* C, and thus assert that the presence of one excludes the other.

A disjunctive proposition, then, presupposes systematic knowledge, and consequently represents a comparatively late stage in the evolution of thought. It is true that disjunction may involve doubt or ignorance regarding any particular individual. We may not be able to say whether A is B or C or D. But before we can formulate such a series of alternatives we must be already acquainted with the whole set of possible conditions, and also with the relation in which those conditions stand to one another. Our knowledge, when capable of being formulated in the disjunctive major premise of an argument, is so exhaustive and systematic that the application to a particular case effected by the minor premise appears almost as a tautology. This will be evident in the disjunctive arguments given below.

There are two forms of the disjunctive syllogism. The first is sometimes called the *modus tollendo ponens*, or the mood which affirms by denying. The minor premise, that is, is negative, and the conclusion affirmative. For example:—

A is either B or C,
A is not C,
―――――――――
Therefore A is B.

The negative disjunctive argument has an affirmative minor premise. It is known as the *modus ponendo tollens*

or the form which, by affirming one member of the disjunctive series, denies the others:—

A is B or C or D,
But A is B,

Therefore A is neither C nor D.

It is of course a very simple matter to draw the conclusion from the premises in these cases. As we have already indicated, the real intellectual work consists in obtaining the premises, especially in discovering the relations enumerated in the major premise. It is in formulating the major premise, too, that errors are most likely to arise. As already pointed out, it is essential that the disjunctive members shall be exhaustively enumerated, and also that they shall exclude one another. But it is not always easy to discover all the possibilities of a case, or to formulate them in such a way as to render them really exclusive. If we say 'he is either a knave or a fool', we omit the possibility of his being both the one and the other to some extent. A great many statements expressed in the form of disjunctive propositions are not true logical disjunctives. Thus we might say, 'every student works either from love of learning, or from love of praise, or for the sake of some material reward'. But the disjunction does not answer the logical requirements; for it is possible that two or more of these motives may influence his conduct at the same time. The disjunctive members are neither exclusive nor completely enumerated.

§ 42. **The Dilemma.** — A dilemma is an argument which includes all possible assertions about its subject-matter under the head of alternatives that involve further consequences, so that one set of consequences or the other must be admitted whichever alternative be allowed. According to the usual definition 'a dilemma is a compound hypothetical syllogism, partly disjunctive in form'. The major premise is always hypothetical, and the disjunction is usually stated in the minor premise. In ordinary life we are said to be in a dilemma whenever there are but two courses of action open to us, and when both of these have unpleasant consequences. In the same way the logical dilemma, when used controversially, shuts an opponent in to a choice between alternatives, either of which leads to a conclusion he desires to avoid.

The first form, called the **Simple Constructive Dilemma,** yields a simple or categorical conclusion: —

> If A is B, C is D; and if E is F, C is D,
> But either A is B, or E is F,
>
> ---
>
> Therefore C is D.

It will be noticed that the minor premise affirms disjunctively the antecedents of the two hypothetical propositions forming the major premise, and that the conclusion follows whichever alternative holds. We may take as a concrete example of this type of argument: —

> If a man acts in accordance with his own judgment, he will be criticized; and if he is guided by the opinions and rules of others, he will be criticized,

But he must either act in accordance with his own judgment, or be guided by the opinions of others;

Therefore, in any case, he will be criticized.

The **Simple Destructive Dilemma** also yields a categorical conclusion. But in this form of the dilemma the major premise has one antecedent and two consequents, and these consequents are denied in the minor premise. The antecedent is therefore denied in the conclusion. A famous example is the argument of Zeno to show that it is against reason to believe that motion really takes place: —

If a thing moves, it must move either in the place where it is or in the place where it is not.

But it cannot move where it is, nor can it move where it is not,

Therefore it cannot move.

It is worth noticing that in this example the minor premise is not disjunctive; that is, it denies the consequents of the major premise *together*, and not disjunctively. All the disjunction here is in the second part of the major premise. The Simple Destructive Dilemma is the only form in which this occurs, and the disjunction may be in the minor premise in this form also.

The hypothetical propositions which make up the major premise of a dilemma do not usually have the same antecedent or consequent, as is the case in the examples just given. When the antecedents and consequents involved are different, the dilemma is said to be complex, and the conclusion has the form of a disjunctive proposition. In

the **Complex Constructive Dilemma** the minor premise affirms disjunctively the antecedents of the major, and the conclusion is consequently affirmative. We may take as an example the argument by which the Caliph Omar is said to have justified the burning of the Alexandrian library: —

If these books contain the same doctrines as the Koran, they are unnecessary; and if they are at variance with the Koran, they are wicked and pernicious,

But they must either contain the same doctrines as the Koran or be at variance with it,

---

Therefore these books are either unnecessary or wicked and pernicious.

A fourth form, the **Complex Destructive Dilemma,** obtains a conclusion made up of two negations disjunctively related, by denying disjunctively the consequents of the hypothetical propositions forming the major premise of the argument. For example: —

If an officer does his duty, he will obey orders; and if he is intelligent, he will understand them,

But this officer either disobeyed his orders, or else he misunderstood them,

---

Therefore, he either did not do his duty, or else he is not intelligent.

By taking more than two hypothetical propositions as major premise, we may obtain a Trilemma, a Tetralemma, or a Polylemma. These forms, however, are used much less frequently than the Dilemma.

The dilemma is essentially a polemical or controversial form of argument. Its object when so used, as we have stated, is to force an unwelcome conclusion upon an adversary by confining him to a choice between two alternatives, either of which necessarily leads to such a conclusion. We sometimes speak of the horns of the dilemma, and of our adversary as 'gored', whichever horn he may choose. Dilemmas, however, like all controversial arguments, are more often fallacious than valid. The minor premise of a dilemmatic argument, as we have already seen, is a disjunctive proposition with two members. But it is very rarely that two possibilities exhaust all the possible cases. The cases enumerated, too, may not exclude each other, or be real alternatives at all. The dilemma is thus subject to all the dangers which we have already noticed in the case of the disjunctive argument. In the minor premise, in addition, it is necessary to see that the canon of the hypothetical syllogism, 'affirm the antecedent or deny the consequent', is observed. Unless this rule is obeyed the logical form of the argument will not be valid.

A dilemmatic argument may be attacked in three ways, the traditional names for which are continuations of the metaphor of the 'horns'.

(1) One may 'escape between the horns'. This is simply to point out that the alternatives presented in the minor premise are not exhaustive, and that there are one or more other possibilities left unmentioned.

(2) The dilemma may be 'taken by the horns'. That is, one may accept the alternative antecedents proposed as

exhaustive, but deny that one or both of the consequents asserted really follow from them. For example: —

> If we have trusts, prices will be excessive; and if we do not have them, our manufacturing industries will fail to meet foreign competition,
> But we must either have trusts or not have them,
>
> ---
>
> Therefore either prices will be excessive or our manufacturing industries will fail to meet foreign competition.

One might reply to this either by denying that there is any inevitable connection between trusts and excessive prices, or by denying that trusts are necessary to enable us to compete with foreign firms.

(3) Sometimes, as a reply to a defective dilemma, a counter-dilemma is proposed, leading to an exactly opposite conclusion. When this is done the original dilemma is said to be 'rebutted'. Whenever such an opposition is possible, each of the two dilemmas by itself fails to state exhaustively either the possible antecedents, or else the consequents following from the given antecedents. Formal rebuttal, therefore, is rather a rhetorical device for showing up the weakness of an opponent's position, than a logical argument for the direct proof of one's own conclusion.

A classical example of such rebuttal is the famous *Litigiosus*. Protagoras the sophist is said to have made an agreement to teach Euathlus the art of pleading for a fee, one-half of which was to be paid to him when he was fully instructed, and the other half when he won his first case in

court. Euathlus put off beginning his practice, and Protagoras finally brought suit for the other half of his fee. Protagoras offered the following argument in his own behalf: —

If Euathlus loses this case, he must pay me, by the judgment of the court; and if he wins it, he must pay me in accordance with the terms of his contract,
But he must either lose it or win it,

Therefore he must pay me in any case.

Euathlus then offered the following rebuttal: —

If I win the case, I ought not to pay, by the judgment of the court; and if I lose it, I ought not to pay, by the terms of the contract,
But I must either win it or lose it,

Therefore I ought not to pay.

The onesidedness of dilemmas which directly confront each other in this fashion is evident in this example. For a complete statement of the case, the major premise of both should be combined. There are really two points of view, or standards of reference, involved in each alike — the expected judgment of the court, and the terms of the contract. Protagoras states the consequent of his first antecedent in accordance with the first standard, and the consequent of the second antecedent in accordance with the second standard. Euathlus simply reverses the application of the standards. But both disputants make use of the two standards alternately, when one only can really

be applied. Either the literal terms of the contract must be observed, and in that case there can be no judgment of the court at all, since the proper ground of action — *i.e.*, Euathlus having won his first suit — is not present. The suit must simply be dismissed. Or else, if a judgment in equity is to be granted, and the contract interpreted in accordance with its spirit and intention, and not with its letter, the appeal is to the judgment of the court on the whole case presented, and this judgment will be either for or against Euathlus. There is therefore no real dilemma involved in the circumstances at all, the appearance of it in each argument being due to the presence of two contradictory points of view.

All dilemmas related in this way of direct opposition, using premises of the same terms, will be found to involve a similar neglect of some aspect of the situation; and this is why we have said that a dilemma in rebuttal, while a striking rhetorical device for attacking an opponent's position, does nothing to establish the truth of one's own. Indeed, if the rebutting dilemma be allowed to remain unsupported by any further argument, it may be considered as presumptive proof that neither party to the debate has any right to a positive conclusion in the matter.

EXERCISES (X)

1. What aspects of thinking are emphasized by the categorical and hypothetical forms of reasoning respectively?

2. How far may the disjunctive proposition be regarded as an expression of ignorance, and what is the justification for the statement that it involves systematic knowledge?

3. In the light of your answers to the two previous questions show how the knowledge expressed in the categorical syllogism is simpler, and less developed, than that expressed in hypothetical arguments, and how the knowledge expressed in disjunctive arguments is more highly systematized.

4. Determine which of the following hypothetical arguments are valid and which invalid; then express the latter in the categorical form, pointing out what are the categorical fallacies which result:—

(1) If a country is prosperous the people will be loyal. The people of this country are loyal and therefore it must be prosperous.

(2) If our rulers could be trusted always to look to the best interests of their subjects, monarchy would be the best form of government; but they cannot be trusted; therefore monarchy is not the best form of government.

(3) If there are sharpers in the company, we ought not to gamble; but there are no sharpers present, so gambling is quite legitimate.

(4) If all philosophical theories were sound, some would be accepted by a majority of thinkers; but as none are accepted by a majority of thinkers, none are sound.

(5) If he had studied his lesson, he would have been able to recite; but he was able to recite, and therefore must have studied his lesson.

5. Criticize the following argument:—

A has either been badly taught or has been himself lazy and indifferent. But as we know that his teacher is not a man of any learning or ability, we may conclude that A is not to be blamed for his failure.

6. How would you attack the dilemmatic arguments on pp. 171 ff.?

7. Discuss the following 'paradox of inference':—

If the conclusion of an inference does not contain something not given in the premises, the inference is useless; and if the conclusion does contain anything not given in the premises, the inference is invalid.

Either the conclusion does contain something not given in the premises or it does not.

Therefore inference is either useless or invalid (Latta & Macbeath).

8. Why is it incorrect to describe a disjunctive syllogism simply as 'a reasoning which has a disjunctive major premise and a categorical minor premise'?

CHAPTER XI

ABBREVIATED AND NON-SYLLOGISTIC FORMS OF DEDUCTION

§ 43. **Enthymemes.** — The term 'enthymeme' seems to have been used by Aristotle for an argument from signs or from likelihood, without complete proof. From this sense of logical incompleteness, the name has come to be applied in modern times to an argument in which some part is omitted. We have already noticed in dealing with the syllogism (§ 10) that one premise is often omitted. Indeed it is but seldom in ordinary reasoning that we arrange our arguments in the strict syllogistic form. We hurry on from one fact to another in our thinking without stopping to make all the steps definite and explicit. We feel it to be a waste of time, and a trial to the patience, to express what is clearly obvious, and so we press on to the conclusion which is, for the time being, the central point of interest.

But the more rapid and abbreviated the reasoning, the more necessary is it to keep a clear head, and to understand what conclusion is aimed at, and what premises are assumed in the argument. To bring to light the hidden assumption upon which an argument is based is often the best means of refuting it.

Enthymemes are sometimes said to be of the first, second, or third order, according as the major premise, the minor premise, or the conclusion is wanting. As a matter of fact, an enthymeme of the third order is a rhetorical device used to call special attention to a conclusion which is perfectly obvious, although suppressed. Thus, for example, 'all boasters are cowards, and we have had proofs that A is a boaster'. Here the conclusion is at once obvious, and is even more striking than if it were actually expressed.

It is usually easy to complete an enthymeme. If the conclusion and one premise are given, the three terms of the syllogism are already expressed. For the conclusion contains the major term and the minor term; and one of these again, in combination with the middle term, is found in the given premise. From these data, then, it will not be difficult to construct the suppressed premise. When the premises are given without the conclusion there is no way of determining, except from the order, which is major and which is minor. It is therefore necessary to assume that they are already arranged in proper logical order, and that the subject of the conclusion, or minor term, is to be found in the second premise, and the predicate of the conclusion, or major term, in the first premise.

§ 44. **Prosyllogisms and Episyllogisms.** — In deductive reasoning it is often necessary to carry on the argument through several syllogisms, using the conclusion first reached as a premise in the following syllogism. For example, we may argue: —

All B is A
All C is B
———
∴ All C is A.
But all D is C
———
∴ All D is A.

It is clear that we have here two arguments in the first figure. The first is called the **Prosyllogism** and the latter the **Episyllogism.** If the argument were carried on further, so as to include three or more syllogisms, the second would form the Prosyllogism with respect to the third, while the third would be the Episyllogism of the second. A concrete example of this kind of reasoning may now be given: —

All timid men are suspicious,
All superstitious men are timid,
———
Therefore all superstitious men are suspicious,
But some educated men are superstitious,
———
Therefore some educated men are suspicious.

It will be noticed that in these examples the argument advances from the premises of the Prosyllogism, to the conclusion of the Episyllogism. It proceeds, that is to say, in a forward direction, developing the consequences of the premises which form its starting point. This mode of investigation is therefore called the *progressive* or *synthetic,* since it goes steadily forward building up its results as it advances. To state the same thing in different words, we may say that the *progressive* or *synthetic* method advances from the conditions to what is conditioned, from causes to effects.

But it is often necessary to proceed in the opposite way. We have often to go back and show the grounds upon which our premises rest, instead of going forward to show what consequences follow from them. And when we do this we proceed *regressively* or *analytically*. To take an example which will illustrate both ways of proceeding: —

> No man is infallible, for no man is omniscient,
> Aristotle was a man,
>
> ---
>
> Therefore Aristotle was not infallible.

In advancing from the premises to the conclusion in this argument our procedure is progressive or synthetic. Instead of reasoning out the consequences of the premises, however, we may go back and show the grounds upon which the major premise rests. It is evident that this premise is itself the conclusion of a syllogism which may be expressed as follows: —

> All infallible beings are omniscient,
> No man is omniscient,
>
> ---
>
> Therefore no man is infallible.

The regressive method goes backward from conclusions to premises, or from the conditioned to its necessary conditions. In scientific investigation it reasons from effects to causes, while the synthetic method advances from causes to effects.

§ 45. **Sorites, or Chains of Reasoning.** — A Sorites is an abbreviated form of syllogistic reasoning in which a subject and predicate are united by means of several intermediate terms. Such a train of reasoning represents sev-

eral acts of comparison, and therefore several syllogistic steps. But instead of stopping to draw the conclusion at each stage, the sorites continues the processes of comparison, and only sums up its results at the close. We may define the sorites, therefore, as a series of prosyllogisms and episyllogisms in which all of the conclusions, except the last, are suppressed. It is usually stated in the following form: —

All A is B
All B is C
All C is D
All D is E
———
∴ All A is E.

It is evident that this train of reasoning fully expressed is equivalent to the following three syllogisms: —

First Syllogism	Second Syllogism	Third Syllogism
All B is C	All C is D	All D is E
All A is B	All A is C (1)	All A is D (2)
∴ All A is C (1).	∴ All A is D (2).	∴ All A is E (3).

There are two rules to be observed in using this form of the sorites: (1) The first premise may be particular, all the others must be universal; (2) The last premise may be negative, all the others must be affirmative. It is evident from an examination of the syllogisms given above that if any premise except the first were particular, the fallacy of undistributed middle would be committed. For in that case the middle term in one of the syllogisms would be the subject of a particular proposition and the predicate of an affirmative proposition. And if any premise but the last

were negative, the major term in the syllogism following that in which this occurred would be distributed in the conclusion without having been distributed in the major premise. We may now give a concrete example of this kind of reasoning: —

Misfortunes sometimes are circumstances tending to improve the character,

Circumstances tending to improve the character are promoters of happiness,

What promotes happiness is good,

Therefore misfortunes are sometimes good.

It will be noticed that the subject of the first premise in this form of argument is taken as the subject of the conclusion, and that the predicate of the conclusion is the predicate of the last premise. This is usually called the Aristotelian sorites. But there is another form which unites in the conclusion the subject of the last premise, and the predicate of the first, and which is known as the Goclenian sorites.[1] This may be thus represented: —

All A is B
All C is A
All D is C
All E is D

∴ All E is B.

Since B is the predicate of the conclusion, the premise in which it appears is always to be regarded as the major.

[1] Rudolf Goclenius (1547–1628), Professor at Marburg, first explained this form in his *Isagoge in Organum Aristotlis*, 1598.

As a result of this, it is to be noticed that the suppressed conclusions in this argument form the major premise of the following syllogism, instead of the minor premise as in the Aristotelian sorites. We may therefore expand the reasoning into the three following syllogisms:—

First Syllogism	Second Syllogism	Third Syllogism
All A is B	All C is B	All D is B
All C is A	All D is C	All E is D
∴ All C is B.	∴ All D is B.	∴ All E is B.

A little consideration of the form of these syllogisms will lead the student to see that the rules given for the Aristotelian sorites must be here reversed. In both forms of the sorites there cannot be more than one negative premise, nor more than one particular premise. In the Aristotelian form no premise except the last can be negative, and no premise except the first particular. In the Goclenian sorites, on the other hand, the single premise that can be negative is the first, and it is the last alone which may be particular.

It is easy to see that we may construct similar chains of reasoning the premises of which are hypothetical. In this case the consequent of each becomes the antecedent of the next, thus:—

If A is B, C is D
If C is D, E is F
If E is F, G is H

∴ If A is B, G is H

or,

G is not H, ∴ A is not B, etc.

§ 46. ***A Fortiori* Arguments.** — There are also a large number of deductive arguments employed in everyday life and in science which are perfectly valid and convincing and yet which cannot, without violent forcing and distortion of natural procedure, be reduced to the syllogistic form. Of these we shall consider in this section a very simple type, and then go on, in a later section, to a more important class.

A fortiori arguments proceed to establish a conclusion by showing that the facts and reasons supporting it are more certain or stronger than those supporting another conclusion that is unquestioned, or generally accepted. They are frequent in dealing with questions of time, space, quantity, and degrees of quality. In fact, we may say that in such matters, whenever the relation involved is not one of contemporaneousness in time, coincidence in space, or equality in quantity, or degree of quality, any argument may be couched in this form. The reason for putting this form into a class by itself is that it is very often employed outside of these fields. To illustrate the two ways in which it is used, for proof and disproof respectively, let us compare a possible argument addressed by a vivisectionist to a meat-eater with one urged upon an anti-vivisectionist by a vegetarian: —

(1)

You admit that it is right to kill and use animals for food,

This is less needful than to kill and use them to discover the causes and remedies of diseases,

---

How much more, then, should you admit that vivisection is right.

(2)

You do not think that it is right to kill animals for vivisection,
Yet this is more needful than to kill them for food,

How much less, then, should you hold that it is right to kill them for food, or, How much more should you deny, etc.

Such arguments as these seem always to involve a comparison of the grounds on which certain conclusions may be justified, when such grounds can be ranked in order of logical cogency. In the one case it is urged that since the reason for the conclusion advocated is stronger than one which it is admitted does establish a certain proposition, the conclusion in question must be regarded as even more firmly established; in the other, as the reason for holding the principle attacked is weaker than that which is regarded as insufficient to justify another principle, it is held that the first principle is still more obviously false than that already denied, or that there is more reason to deny it than there is to deny the other. Hence the name *argumentum a fortiori*, 'argument from, or by, the stronger', ('reason' being understood).

§ 47. **Systematic Deduction.** — Many times in the course of our analysis of deductive inference we have had implicitly or explicitly to indicate the limitations and defects of the syllogism as a form of reasoning. The traditional categorical syllogism, as we saw, is essentially a process of subsumption, of tracing the relations of inclusion and exclusion among wider and narrower classes of objects. It naturally belongs to that stage of our knowledge in which classification is the goal of investigation. Reality, on this

view, is regarded as made up of relatively self-enclosed things and their qualities, and of separate and distinct subjects qualified by predicates representing natural kinds or classes. Recognizing no other possible type of knowledge, Aristotle made the excusable mistake of regarding all demonstration as syllogistic in nature.

Consideration of hypothetical and disjunctive arguments, however, carried us beyond this point of view, for they revealed the presence of a new ideal of knowledge, namely that of the systematic interconnection of things. It has become increasingly evident with the growth of modern science that the objects and phenomena of our experience are bound together in systems. And furthermore, analysis of some of the simplest of these systems has revealed another type of proposition, sometimes designated as 'relational'.[1] That is to say, these propositions give expression primarily to the relations of objects to each other, within a system, rather than to the relation of an object to its qualities, or to a class under which it may be subsumed.

Now the important fact to be noted here is that arguments, inferences, involving such propositions are not syllogistic in form. For example, the inference,

$$\begin{aligned} A &= B \\ B &= C \\ \therefore A &= C, \end{aligned}$$

while resembling a reasoning in Barbara, is not really reducible to that form. For (1) the propositions of which it

[1] See above, pp. 74 ff.

is composed have neither subject nor predicate in the traditional sense of those words. (2) There is no middle term; and (3) the number of terms is more than three. The reader can readily construct for himself similar deductive arguments, involving numerical, spatial, temporal, family, or causal relationships, which violate the formal rules of the traditional syllogism, but nevertheless yield perfectly valid conclusions. Or consider the example already cited (p. 165), concerning the relation of bodies to each other according to the law of gravitation. This law is a principle of systematic organization *according to which* all material particles in the physical universe are interrelated. If we are given also certain characteristics of some of these elements, we can infer (deductively) the corresponding characteristics of other elements. Adams and Leverrier were reasoning in this way, for example, when they inferred from the movements of Uranus and its place in the system of gravitating bodies, that there must be a body, not yet observed, exerting an influence upon it. The discovery of Neptune was the result.[1] When we come to study the inductive aspect of inference, we shall see that there, too, the idea of system is all-important.[2]

In the following passage Latta and Macbeath succinctly distinguish that type of reasoning exemplified in the simple categorical syllogism from what we have ventured to call systematic deduction. Systematic deduction proceeds "on the fact that everything is an element in a system and has its characteristics determined by its relations to other ele-

[1] See, below, pp. 303 ff.

[2] See, *e.g.*, pp. 241 ff., 369 ff., 384 ff.

ments within the same system. Syllogistic inference proceeds on the fact that many individuals in nature are of the same kind and that what is true of the kind is true of each individual." Accordingly, all that the categorical syllogism can predicate of the individual is "what is true of the kind or universal of which it is an instance. It cannot show how the universal differentiates itself in the particular case. It predicates the same character of each of the individuals of a kind, and therefore it cannot determine the individual characteristics of any. While what is true of all the others is true of this one, it is not enough to distinguish it from any of the others. The knowledge of the individual that we get by syllogistic inference is thus very indefinite. If we want to infer the distinguishing characteristics of an individual, we must find a system of which it is an element, and infer them from its relations to other elements within the system." [1]

We should not, however, conclude from such considerations as this that syllogistic reasoning and systematic deduction are radically different in kind. Rather, the former is included as a very simple case within the latter. But due to its extreme simplicity the categorical syllogism yields only a very inadequate grasp of the systematic interconnections of things. These interconnections, when adequately traced out, are found to be mutually reciprocal, so that within a system, as represented, for example, in a disjunctive reasoning, it is possible to infer from one premise and the conclusion to the other premise as well as from the

[1] *The Elements of Logic*, pp. 231, 232.

original premises to the conclusion. In the simplest type of syllogism, on the other hand, the relation between the premises and conclusion is not thus reciprocal. The premises support the conclusion, but the truth of the conclusion does not necessarily imply the truth of the premises. And these distinctions are the more important in view of the fact that so small a part of our deductive reasoning, either in ordinary life, in the law courts, or in science, finds adequate expression in the categorical syllogism.[1]

Now because of the fact that the truth of the conclusion of a syllogistic demonstration does not necessarily imply that of the premises, traditional formal logic was faced with the double problem, how to acquire true premises, and how to test their truth once they were acquired. Of course the premises of any given reasoning might have been conclusions of preceding arguments, but to go on indefinitely in this way would be to become involved in an infinite regress. Aristotle's own solution of the problem was that the original premises of demonstrative (*i.e.*, for him, syllogistic) arguments were arrived at by induction, and, once obtained, were grasped by intuition as self-evidently true.[2] Unfortunately, however, Aristotle himself had little to say about the process of induction and its relation to deduction. More unfortunately still, later logicians became divided on these questions. Some of them, as we have seen (Ch. II, above), came to regard deduction as the only valid form of

[1] See Bosanquet, *Implication and Linear Inference*, for further discussion of the relation of the syllogism to deductive inference in general.

[2] See the final chapter of his *Posterior Analytics*, and see below, pp. 386 f.

inference, while others looked upon induction as of vastly more importance, maintaining that new knowledge could be attained only through employment of the so-called inductive methods. The former argued that the ultimate premises were self-evident because innate, present in the human mind from the very beginning. The latter denied the existence of any such innate truths, maintaining instead the ambiguous principle that 'all knowledge is derived from experience'. Consistently with this doctrine, they denied that any real knowledge is absolutely certain; it is merely more or less probable.

The conflict between these two schools of thought has gone on for centuries — more than long enough to show that there is no way out of the impasse once granted the fundamental dichotomy, upon which it is based, between 'experience' and 'intuition'.

Now it is a virtue of the conception of knowledge as systematic that it definitely puts an end to this struggle, and shows us how to solve the problems from which it issued. In this connection we shall make use of the idea of development, the import of which, for logic, was briefly indicated in the second chapter of this book.

If, as a matter of fact, we observe the progress of any science or group of sciences, we shall find that what actually happens is this. As our knowledge becomes more complete, the elements, the particular facts, belonging to any field of investigation come to be more and more closely interrelated. "Each new element whose place in the system we discover enables us to see the nature of the system as a

whole more clearly. . . . Our insight into the nature of a whole and our insight into the character and interconnection of its parts proceed together. . . . We see more clearly the kind of system the physical world is, as science brings to light the exact quantitative relations between its parts. We grasp the nature of the solar system, as astronomers exhibit the mutual relations of its elements in the way of gravitation, etc. And at the same time and by the other side of the same process, we see more clearly the character and mode of behavior of the constituents or elements of these systems. Our knowledge of the system and our knowledge of its elements develop *pari passu*."[1] And we infer, inductively or deductively, according to the nature of the system, from part to whole, from part to part, or from whole to part, within it. Thus both 'premises' and 'conclusions' undergo modification in the course of the developing process which the acquisition of new knowledge illustrates. New premises are got by induction, as Aristotle taught, but their validity is insured, not by a sudden flash of a mysterious intuition, but rather by their power to unify hitherto disconnected facts. And new knowledge is indeed derived 'from experience', but from an experience conceived of as including all that we already know. From this point of view the contrast indicated by the misleading distinction between knowledge that is absolutely certain and knowledge that is only probable may be stated in a manner more in keeping with the actual status of scientific judgments. Once grant that no one proposition stands on its

[1] Latta and Macbeath, *op. cit.*, pp. 243, 244.

own feet but belongs to a system, apart from which its real meaning cannot be grasped, and obviously the degree of truth to be ascribed to it is a function of the system to which it belongs. If the system meets such obvious and time-worn tests as those of consistency, simplicity, and comprehensiveness, the propositions composing it will be accepted as true, subject only to future developments within the field in question. For example, $F = MA$ is a proposition which has long been accepted as true, but since the time of Einstein it has undergone a radically new interpretation. The Theory of Relativity has given it a new context, which has changed its meaning. And this is the fate in store for almost all scientific propositions — to be subject to a constant process of reinterpretation as scientific knowledge develops.[1]

It is quite impossible to formulate specific rules, analogous to those applicable to elementary syllogistic inference, outlining in detail the course of deductive reasoning in all such systems, for the simple reason that their number is legion. The detailed rules governing the process of inference in the case of the system of family relationships, for example, are quite different from those applicable to the case of spatial relationships. What these rules are can be determined only in the light of knowledge of the respective systems in question. Thus, for example, I can infer from the premises 'A is the child of B' and 'B is the child of C' that 'A is the grandchild of C' only if I have sufficient knowledge of the general nature of family relationships. The wise counsel of

[1] See further below, pp. 392 ff.

General Pershing to his troops before going into battle applies equally well to intellectual undertakings. "Let us not for a moment forget," he said, "that while study and preparation are necessary, war itself is the real school where the art of war is learned . . . you must learn in the actual experience of war the practical application of the tactical principles that you have been taught during your preliminary training. . . . Whatever may be the changing conditions of war, those principles remain practically the same, and you should constantly bear them in mind. Now that you are going to take a place in the line of battle you will be called upon to meet conditions that have never been presented to you before. When confronted with a new situation, do not try to recall examples given in any particular book on the subject; do not try to remember what your instructor has said in discussing some special problem; do not try to carry in your minds patterns of particular exercises or battles, thinking they will fit new cases, because no two sets of circumstances are alike; but bear in mind constantly, revolve in your thoughts frequently and review at every opportunity those well-established general principles so that you may apply them when the time comes." In other words, what we can derive from a study of deductive reasonings is, not a ready-made set of rules to fit any special case of reasoning, but rather an understanding of the general principles implicit in all of them alike, and most simply illustrated in the case of the categorical syllogism.[1]

[1] See pp. 45 ff. above, and Part III, below.

PART II

INDUCTIVE METHODS

CHAPTER XIII

THE PROBLEM OF INDUCTION

§ 52. **The Problem of Induction.** — In Part I we have studied the general nature of deductive inference, and have learned what conditions must be fulfilled in order to derive valid conclusions from given premises. But the question how the premises themselves are established was not discussed. It is true that the premises of one syllogism are sometimes proved by means of a Prosyllogism, and that it may be possible to find in turn general propositions to support the premises of this latter argument. But somewhere this process of formal proof must have an end.. At last we reach propositions concerning which we can say only that their truth is guaranteed by experience. It is from experience that propositions are obtained like 'man is by nature a social being', 'water is composed of hydrogen and oxygen', which serve as the premises of syllogisms. To say that these propositions are learned through experience does not mean however, as we have seen (p. 194), that they have been obtained without thinking. For to experience is not merely to feel or to have sensations; it is also to put things together, to interpret, to appreciate to some extent what our sensations stand for and signify. When I say 'yonder tree is an elm', this proposition is the outcome of my own thinking; it is my interpretation, on the basis of

past experience, of certain sensations of color and light and shade, together, it may be, with certain muscular sensations from the movements of the eyes. Our thought is constantly bringing new sensations and perceptions into relation with former experiences, and in this way building up and organizing our world of knowledge. To interpret the real world — not only the physical world, but the psychological and the social world as well — is then the business of thought, and this, as we have seen, is to relate the new in some way to what we already understand. Our sense-perceptions, just as they come, are without order or system. Think, for example, of the various things you are sensing at the present time. The greater part of these are not consciously attended to or thought about; they are taken for granted or roughly classified on the basis of some past experience. But if one is really thinking there is some fact or relation that is taken as a problem, and for which one is seeking an interpretation, *i.e.*, some way of thinking this fact or relation that will bring it into place and adjust it to what is already known.

Apart from this task of interpreting the real world, thought has no function, and does not exist. Deductive reasoning is not a distinct and separate kind of thinking, but is a necessary part of the work of building up our knowledge of the world in systematic form. Without thinking, then, no knowledge, no real experience. But we must remember that thinking is no mere play of ideas in our heads. It exists only in relation to what is objective and real. In a certain sense it always goes back to a *datum*, to percep-

tion. Kant's famous saying that 'perceptions without conceptions (*i.e.*, thoughts) are blind, while conceptions without perceptions are empty', is well worth remembering.

The problem of *Induction*, with which we are primarily concerned in this part of the book, is how we are able to derive from experience general propositions or principles. It is on these, as we have seen, that we base our conclusions in deductive reasoning. The difficulty is that experience seems to give information regarding individual things and their qualities only. One learns by experience the qualities of this rose, or of this piece of iron; but how is one to discover the general nature of the rose or of iron as such? As a matter of fact we are constantly deriving general statements from individual experiences; and in doing this we usually bring up, in a more or less systematic way, a number of cases or instances and use them as the basis of the general statement. And this process of *generalization*, or passing to a general conclusion on the ground of certain instances or cases that have been advanced, may be called *Induction* (ἐπαγωγή). This definition is of course only preliminary, and does not attempt to distinguish valid and invalid induction. We have to go on to consider more in detail both the conditions necessary to render the process valid, and the meaning of the generalization at which we arrive.

§ 53. **The Enumeration of Instances.** — In the first place Induction is not the outcome of a complete enumeration of instances; but from an examination of a certain number we *infer* the general mark or principle that is involved in all the instances. Where all the instances have been examined, the

result may be summed up at the end in a proposition that is universal in form; but in such a case there has been no Induction, no passage to any truth that is really general. For example, after measuring each individual in a company and finding that A is less than six feet in height, B less than six feet, and so on for the rest, I might make the assertion, 'No one in this company is more than six feet tall'. This however would be nothing more than a summation of results, and not a genuine Induction at all. Nevertheless some writers regard such procedure, where all the instances are examined, as the only perfect form of induction. Thus Jevons says: "An Induction, . . . is called Perfect, when all of the possible cases or instances to which the conclusion can refer have been examined and enumerated in the premises."[1] On the other hand, where it is impossible to examine all the cases, the inductive process is regarded as Imperfect by the same writer, and the conclusion expressed in the general law as only probable.

Now this view, though mistaken, is interesting because it assumes that it is the business of Induction to count instances. When it is possible to examine all the cases we can have certainty; when this is impossible (as is usually true), the unexamined instances have to be regarded as more or less probable only. No other conclusion is possible so long as we merely enumerate or cite instances without attempting to analyze them. A mere *factual* connection of two events, P and Q, though experienced a thousand times, does not warrant the universal proposition, 'All P is Q'. As a

[1] *Elementary Lessons in Logic*, pp. 212–213.

matter of fact scientific induction always does get beyond a mere citation of unanalyzed instances. "Induction which proceeds by merely citing instances", says Bacon, "is a childish affair, and being without any certain principle of inference it may be overthrown by a contradictory instance. Moreover, it usually draws the conclusion from too small a number of instances, taking account only of those that are obvious."[1] This is an excellent description of the popular unscientific way of seeking to establish universal connections between events, by citing random instances where the events have happened to be found together. It is generally easy, for example, to cite instances where dreams have come true, or where one member of a dinner party of thirteen has died within a year. This species of Induction is, as Bacon says, "*res puerilis*", since it simply *asserts* the connection without justifying it or making it intelligible, by bringing to light any principle of coherency. The possibility of contradictory instances is not excluded, and the cases cited lack definiteness and precision, no account being taken of the attendant circumstances and conditions.

It should be clear, on reflection, that scientific Induction aims at establishing a universal law that does not refer primarily to cases or instances at all. And the method it employs, as will be shown later, is to discover the law by analyzing the instances and reading it out of them, rather than by merely summing them up. When I conclude inductively that 'sentimental people are selfish', or that 'the maple has a forked fruit-key', the universal statement

[1] *Novum Organum*, Bk. I, Aph. CV.

is not to be taken as merely summing up instances. Such propositions are rather assertions about universal types or kinds — the nature of sentimental people as such, or of maple trees as such. What has been established, granting that the induction is valid, is a coherence of characters forming a kind or type, so that the conclusions might be expressed in hypothetical form: 'if sentimental, then selfish', 'if a maple, then a forked fruit-key'.

To discover such universal principles of connection through the analysis and comparison of instances is, then, the goal of what may be called Scientific Induction. But we may also speak of Enumerative Induction as a lower and less complete form. In practical life we often depend with confidence on a conclusion based on a somewhat careful survey of instances. It is of course easier to rest on the authority of the instances, taking the connection as a fact, than to set systematically to work to analyze the instances in a scientific way in order to determine exactly the universal form of the law. It is likewise clear that these unanalyzed or only partially analyzed instances form the starting-point for scientific induction; and that therefore Enumeration must often play an important part in the preliminary stages of an investigation. But in certain fields of investigation we have to go on counting instances because there seems to be nothing else to do. We simply find P and Q invariably conjoined as a fact in experience, but are unable to analyze out the conditions and so either *mediate* the connection, or exhibit the precise form of the law. We cannot get a genuinely universal proposition as-

serting, 'P as such is connected with Q as such', or, 'if P, then Q'. But the Enumerative conclusion simply affirms that all instances of P (so far as experienced) are connected with Q. Nor is the particular nature of the connection defined in this form of Induction. P and Q, for example, may be connected directly, or in some indirect way, as through a common dependence on some third thing, M. In the next chapter something further will be said of Enumeration, and how it may contribute, when used intelligently, to the ends of scientific Induction. Considered in itself, however, as dealing merely with instances, we see how far it falls short, both in certainty and exactness, of the ideals of scientific knowledge.

§ 54. **Induction through Analysis.** — Scientific Induction, then, aims at discovering some typical character or law of behavior. This usually requires the examination of a considerable number of instances. But the general proposition is not obtained by simply counting the instances, or by adding them together. The purpose of taking a number of instances is to facilitate analysis, to aid us in eliminating characters or circumstances that are accidental or irrelevant, and at the same time, through these exclusions, to exhibit and define more clearly the essential character and relations of the subject we are investigating. The process of analysis is thus at the same time a process of synthesis; the process of excluding the irrelevant, a process of defining the essential. But it should be noted that if the instances are to lead to this result they must, so to speak, be selected for this purpose. They are not likely to be in-

structive, if they are chosen at haphazard. If the instances were all alike, for example, we should not gain anything by adding to their number, or if we could discover nothing in common among them, we should not be likely to select them. It is clear, then, that instances, to be instructive, must be *selected* with reference to the purpose of the investigation, and that the work of selecting instances is an essential part of the work of induction. It is with this end in view that we extend our observations over as wide an area as possible, drawing instances from different parts of the field. In natural history, for example, specimens are taken from different localities in order to determine by comparison what features are specific or generic characters, and what mere 'local variations'. What we seek to obtain is not merely a *number* of instances, but instances which show *differences that might be significant for our problem.* What differences or circumstances might be significant, we cannot of course know in advance. We can only guess, guided by our past experience, what might make a difference, and hope, by drawing instances from different parts of the field, to include all the significant circumstances. The function which the instances when thus selected fulfil is of course to exhibit what is essential by eliminating circumstances which are, for the purposes of the investigation, superfluous and irrelevant.

Experimentation, when it is possible, is another way of performing the same work of analysis and elimination. Hence in fields where experiments can readily be made, Induction does not have to depend upon an assemblage of

instances. The experimenter, having control of the conditions, can produce the variations he wishes to observe, changing one thing at a time and noting the result. In this way he is able to strip the phenomenon of superficial features that are connected with it only accidentally, or in a particular case, and by so doing lay bare its universal properties and modes of acting. But in experimenting, just as in collecting instances, there must be a guiding idea or purpose. In both cases alike, information is gained only by having questions or provisional guesses in mind, and then selecting for observation what is necessary to enable us to decide which guesses are false and which true.

What guides the selection of instances in an inductive inquiry and also determines the character of the experiments to be performed, is the tentative conception or hypothesis which the investigator has in mind. We must look, both in collecting instances and in setting up experiments, for facts which are significant, that is, which will help to answer the questions we have in mind. Bacon discusses at length, and classifies under twenty-seven different heads, what he calls Prerogative Instances, which, as especially instructive, should be the first and last objects of our investigation. Some of his headings are: 'Solitary instances', 'migrating instances' (where the phenomenon is in process of coming into existence or disappearing), 'clandestine instances', 'deviating instances' (as sports, or pathological cases), 'bordering instances', and 'crucial instances'. This last name (*instantia crucis*) is drawn from the metaphor of the cross erected where two roads meet to indicate the different

directions. When we have alternative conceptions or explanations in mind, either of which appears possible, we look for some crucial instance, or devise some crucial experiment that will point the way by eliminating one of the alternatives.[1] To know what facts would really be crucial in any given case, it is of course necessary to have some definite and systematic knowledge of the field in which the phenomenon under investigation falls. Only when this condition is realized are we able to interpret rightly the bearing of the new instance or experiment on our problem.

The process of Induction, then, might be represented in the form of a Disjunctive Syllogism, where the conclusion is reached by eliminating successively all but one of the Disjunctive members. For example: —

> This phenomenon, P, is either A, or B, or C.
> These facts prove that it is not A; and these that it is not B.
> ---
> Therefore P must be C.

This account is fundamentally correct in principle, though the Disjunctive Syllogism represents the process as more formal than it really is. It is not to be supposed that at the beginning of an inductive investigation all the possibilities are definitely and disjunctively formulated. The various possibilities, and their relation to one another, rather come to light as the examination and analysis proceed. And the conclusion is never *merely* the result of the process of exclusion. In other words, we do not accept C merely

[1] Examples of crucial experiments may be found among the miscellaneous exercises at the end of this volume.

because we cannot think of anything else; but through the process of excluding A and B, C has become, to some extent at least, positively defined and determined. In dealing with any real problem we cannot make any significant denial without thereby implicitly affirming and defining something else. These considerations will come up for discussion again, particularly in Chapter XVII, where an account is given of the more explicit use and nature of hypotheses. In the meantime however the disjunctive principle may be regarded as the working basis of inductive procedure, though, especially in the earlier stages of this process, the disjunctive members are not formally enumerated, or set over against one another as exclusive possibilities.

Where now, we may ask, do the conceptions which are thus put forward in more or less definitely disjunctive form, and tested by means of instances and experiments, have their source? They arise in the mind itself, and are expressions of its own theorizing activity. These conceptions, however, are not mere uninstructed guesses, but are formulated in the light of the knowledge already achieved. Induction, as a scientific process, bases itself on the relations and distinctions that are found in ordinary experience, and simply carries these farther and makes them more definite and consistent. Now in the language of ordinary life there is already given a preliminary classification and arrangement of the fundamental aspects of experience. In ordinary speech and in everyday practical relations there is present a certain organization of experience. And it is this which is taken as the starting-point for the scientific interpretations

which are to correct and extend the old. The phenomenon that we set out to interpret can only be understood in the light and with the help of what is already assumed as known. It is because we are able to perceive or imagine the likeness of the new to something with which we are already familiar that it is possible to think of it in relation to the rest of our experience. If any phenomenon were to appear as absolutely unclassifiable, or totally unlike anything ever experienced before, there would be no means of getting hold of it, so to speak. And just because it might be anything, it would be for us as good as nothing. Even to attend to it would be impossible, for attention involves comparison. But the truth is that new facts and experiences always appear as modifications or variations of existing experience. In other words, although they have the element of unfamiliarity, it is yet always possible to discover in them some point of resemblance or identity with what has gone before. This resemblance or analogy in certain respects with what is already familiar leads us to assume that they may be of the same general type or kind as the latter, and that they will be found to have similar properties or modes of operation. But this is as yet only an assumption that must be tested before being accepted as true. Further analysis may show that this assumption is based on a mere surface resemblance which does not warrant the interpretation made. Or, as is more usually the case, examination may disclose analogies which only allow the phenomenon to be classified as belonging to this or that general field. But the point to be noted is that through analogy its sphere has

been determined. There are now only a definite number of possible interpretations, which take more or less definitely the form of a disjunctive proposition; P falls in the general field M, and is therefore A or B or C. Each member is put forward on some positive ground, and is thus a genuine possibility, not a mere unsupported guess. But it is only a possibility — something whose truth is still to be determined — and so its function is to operate as a plan or schema, pointing the way to further examination and testing through new instances and observations.

Our discussion has accordingly shown that Induction is able to pass from instances to a general conclusion only when the instances are selected because of their bearing on conceptions and hypotheses with which we are experimenting. Moreover, in forming these tentative hypotheses, we are guided in the first place by the analogy of the phenomenon under investigation to what is already known. Analogy and Hypotheses are then indispensable in Induction from the beginning, though the account of the more formal and explicit use of these operations is postponed to the later chapters.

EXERCISES (XIII)

1. Give a statement of the general problem of induction. What precisely is the problem in understanding how the mind reaches universal truth? Explain.

2. It has been said that deductive logic seeks to bring ideas into harmony with each other, and inductive logic to bring ideas into harmony with facts. Comment upon this distinction.

3. "Scientific induction aims at establishing a universal law that does not refer primarily to cases or instances at all" (p. 231 of the

text). How can this be true if induction be a process of generalization "on the ground of certain instances or cases that have been advanced"?

4. What rules can you formulate for the selection of instances in an inductive investigation?

5. Why is it wrong, in principle, to regard Elimination as the essential principle of induction?

6. It is sometimes said that all inference is essentially deductive. If this be true, does it follow that there can be no such thing as induction? Explain.

CHAPTER XIV

THE ASSUMPTIONS OF INDUCTION — STAGES IN THE INDUCTIVE PROCEDURE

§ 55. **The Assumptions of Induction.** — It is part of the task of Logic to make us conscious of the assumptions of our thinking. We have found in dealing with syllogisms that it is often necessary to look for the premise or principle assumed in drawing the conclusion. But in addition to these special assumptions which are taken as the basis of argument in particular cases, there are more general assumptions made by each science in the very process of defining its own standpoint and working conceptions. Moreover, still more general assumptions may characterize *groups* of sciences, as for example the natural sciences, the historical sciences, etc. Finally the question may be raised as to what is assumed in *all* thinking — what are the universal assumptions of thought — and what form these assumptions take in Induction. In § 9 we spoke of the Laws of Thought, and under the names of Identity and Non-Contradiction reference was made to the principles of consistency on which deductive logic is based. Now since Induction and Deduction, as both processes of reasoning, are different rather in form than in fundamental character, their assumptions are not unrelated to each other. Indeed, the assumptions of Inductive thinking are more concrete expressions of the laws

of thought than are the formal expressions of Identity and Non-Contradiction mentioned in connection with the syllogism.

What we appear to assume in inductive reasoning is that the reality with which thinking is dealing is systematic and coherent. There is no direct method of proving that the world is not composed of a collection of particular things resembling one another more or less in an accidental or external way, but at bottom having nothing to do with one another. The only proof is that it would be impossible either to understand or to deal practically with such a world. For it would be a world in which experience could teach us nothing, since events might happen in any order or in any way, and it would never be possible to infer anything. We assume, therefore, and must assume, that the world is a cosmos, not a chaos. And this means that there are universal relations and connections of events which, if once discovered in their true nature, may always be depended upon. 'What is once true is always true'. A (*e.g.*, the properties of iron, or the principles of heredity), once accurately determined and defined, is A, however various may be the instances in which it appears. To say, as is sometimes done, that in Induction it is assumed that what is true of certain instances will be true of all other instances resembling these, is not entirely accurate. For as we have seen, genuine induction is not based on instances at all, but on the discovery through analysis of a typical nature or law of action. What our thinking assumes is that identity of law and identity of nature exist in and through the diversity of things, and that

it is in virtue of these universal principles of connection that the world is a coherent and intelligible system. Induction is only possible on the assumption that things not only *are* together but *belong* together. On this assumption it has to work out the special mode of 'belonging' in various fields of phenomena; to bring to light the identity of nature or law that connects things which at first sight appear diverse and unrelated.

The question of how this identity of nature, connecting things, is to be conceived, is a very fundamental one, both in science and philosophy. We have already seen that to discover a genuine identity it is necessary to penetrate beyond striking resemblances and superficial sense qualities to some deeper-lying nature. Moreover, the universal nature of a thing cannot be discovered in the form of some essence or substance that remains permanent and unchanging. It must rather be conceived dynamically, as a mode of activity, or rather as a system of activities in which all the parts are involved, and through which they are correlated. And furthermore, the activity of a thing, which constitutes its nature, carries it, so to speak, beyond its own boundaries. It acts upon other things, and is in turn influenced by them. Its so-called properties are statements of its relations to other things. It cannot therefore be conceived as an isolated, unchanging essence, but must be defined through the constancy of behavior shown in its changing relations to its environments. For example, the universal nature of man is not found in some unchanging substance, either material or spiritual, that inheres in the

different human individuals. It consists rather in the system of functions, physical and mental, through which he expresses his relation to the world of persons and things. Nor, in the case of man, are the activities constituting his nature modes of reacting with unvaried uniformity, but functions of adjustment and organization which develop in the light of the work they are called upon to perform.

§ 56. **Stages in the Inductive Process.** — Induction we have already seen to be a process of interpreting facts in terms of general conceptions or principles. This description would however apply equally well to Deduction; and as a matter of fact these are not different *kinds* of thinking, but different *methods*, which are necessary to supplement each other in the task of making things intelligible. The various sciences have to start with particular facts learned through experience. The knowledge of general laws and principles comes later, and is derived from a study of the particular facts. It is clear, then, that the procedure of all the sciences must be inductive, at least in the beginning. The various sciences are occupied, each in its particular field, in the task of discovering order and relation among phenomena that at first sight appear to be lawless and disconnected. But in carrying out this undertaking our thinking uses every means that will help it towards its desired end. It is often able, after pushing inductive inquiries a little way, to discover some general principle, or to guess what the law of connection must be. When this is possible it is found profitable to proceed deductively, reasoning out what consequences necessarily follow from the assumption of such a

general law. Of course it is essential to verify results obtained in this deductive way by comparing them with facts as actually experienced. The truth is that it is impossible, in actual thinking, to separate induction and deduction: the two processes constantly go hand in hand and are mutually supplementary.

Again, it must be remembered that the inductive process, considered broadly as the progressive interpretation of experience, is continuous throughout. What is already known is always taken as the starting-point for a new investigation. And although the immediate purpose of any special inquiry may soon be satisfied, the results obtained lead to new questions which can be answered only by further analysis and investigation. There is then no break — no fundamental separation — between the facts with which induction starts and the more highly developed theories and generalizations it is sometimes able to reach. What we call facts are themselves the results of former processes of thinking and interpretation, as well as the starting-point for new analysis and theorizing. There is a constant passage from one stage to the other, theories when approved and generally accepted coming to be regarded as facts, and facts when critically examined disclosing the theoretical basis on which they rest. For example, we say that it is a 'fact' that the earth revolves on its own axis. Yet this, not very long ago, was regarded as an 'incredible hypothesis'. And when we reflect we see that this 'fact' is really a conception — or a part of a system of conceptions — enabling us to bring together in our thought a number of simpler 'facts'. And

these latter, if examined, would in turn prove to be constructed by coördinating and generalizing still simpler data, the truth being that all facts involve ideas.

Whewell has spoken of Induction as "the true colligation of facts by means of an exact and appropriate conception"; and he goes on to point out that the distinction of fact and theory is only relative. "Events and phenomena considered as particulars which may be colligated by Induction, are facts; considered as generalizations already obtained by colligation of other facts, they are theories." [1]

§ 57. **Observation and Explanation.** — The inductive process being thus continuous, how are its different stages to be distinguished and classified? We may still adopt the customary terms, and speak of Induction as including both **Observation,** or **Description,** and **Explanation,** though it must be remembered that the one process really involves the other. Sometimes the relation between Observation and Explanation is stated in quite a misleading way. It is said that in undertaking an investigation we must observe and describe the facts as accurately as possible, and only after this is done proceed to theories and explanations. Now as has been shown, this is to make an artificial separation between collecting and describing the facts, and relating or explaining them. As we have seen, both processes go on simultaneously. The observation of instances presupposes some guiding idea, some provisional hypothesis, perhaps held in the mind as a question to be answered. We discover the relevant facts as we go along with our investigation,

[1] *Novum Organum Renovatum*, Bk. II, Aph. XXIII.

just as we discover the appropriate conception or explanation. And just as the facts observed and described involve theories and conceptions, so the explanation to which we proceed is simply a fuller and more accurate description. When the close and necessary relation of these stages of induction is kept in mind, there is however some advantage in maintaining the distinction between Observation of the nature of particular facts and the wider organization of facts and relations effected by what we call Explanation.

It is the business of the former process to employ various methods and devices in order to determine as accurately as possible the nature of the starting-point. It is essential to have a full and accurate survey of the terms of the problem, and to note carefully every clue that may lead to its solution. In the first place, the different *qualities* of things must be accurately observed and distinguished. But accurate observation in science leads almost directly to the determination of *quantitative* relations through measurement. Under this head fall processes of enumeration, the measurement and recording of space and time relations, the determination of weights, and the measurement of the so-called secondary qualities like heat, sound, and color. The special technique through which such observations are carried out and rendered precise in the different sciences must be learned through occupation with the actual phenomena. In each science questions arise regarding methods of measurement — the determination of the units to be employed, means of measuring indirectly when direct measurement is impossible, the most accurate method of summing up observations

and of eliminating errors — as well as problems regarding the most convenient means of representing quantitative relations through mathematical formulæ, graphs, etc. In addition, the use and manipulation of various instruments designed to supplement and render more accurate the observations of the senses have to be learned; the fingers often require to be trained to perform delicate operations; and a special education of the senses and attention is necessary in some fields before results of scientific value can be obtained. This technical knowledge and skill in the employment of the instruments and methods of observation and description within any science is to be attained, as already stated, only by actual practice. We distinguish practically this work of collecting data — which may be extended over months or years — from the construction of the explanatory theory, the former often seeming to demand the power of patient observation and skill in mechanical manipulation rather than logical reasoning.

It is important, however, to remember that scientific observation itself involves intellectual activity. To observe — at least in the sense in which the word is used in scientific procedure — requires something more than the passive reception of impressions of sense in the order in which they come to us. Without some activity on the part of mind it would be impossible to obtain even the imperfect and fragmentary knowledge of everyday life. But accurate observation is one of the means that science employs to render this knowledge more complete and satisfactory; and when observation thus becomes an exact and conscious

instrument, it involves, to even a greater extent than in ordinary life, intellectual activities like judgment and inference. It is because this is true, because scientific observation demands the constant exercise of thought in selecting and comparing the various elements in the material with which it deals, that it affords such excellent intellectual discipline. The observational sciences do not merely train the sense-organs; the discipline they afford is mental as well as physiological, and it is of course true that mental training can only be gained through the exercise of mental activity.

It is quite true that it is of the utmost importance to distinguish between a fact, and further inferences from the fact. As will be pointed out in the chapter on Inductive Fallacies, errors very frequently arise from confusing facts and inferences. This does not mean, as we have seen, that facts exist apart from theories. But in any particular case if we would avoid confusion we must distinguish sharply between the data and further constructions to which we proceed. Especially important is it not to confuse facts with fancies, or with judgments motived by subjective feelings. The point emphasized in the previous paragraph, however, is that it requires a certain amount of *thinking* in order to get a fact at all. Facts do not pass over ready-made into the mind. Simply to stare at things does not give us knowledge: unless our mind reacts, judges, thinks, we are not a bit the wiser for staring. To observe well it is necessary to be more or less definitely conscious of what one is looking for, to direct one's attention towards

some particular field or object; and to do this implies selection among the multitude of impressions and objects of which are we conscious. Moreover, scientific observation requires analysis and discrimination. It is not unusual in text-books on logic to symbolize the various facts learned through observation by means of letters, *a*, *b*, *c*, etc., and to take it for granted that they are given in our experience as distinct and separate phenomena; but as we have just seen, judgments of analysis and discrimination are necessary to separate out the so-called 'phenomena' from the mass or tangle of experience in which they were originally given. Again, to determine the nature of a fact through observation, it is essential to note carefully how it differs from other facts with which it is likely to be confused, and also, to some extent, what relations and resemblances it has. But such knowledge presupposes that thought has already been at work in forming judgments of comparison.

A distinction is sometimes made between observation and experiment. In observation, it is said, the mind simply *finds* its results presented to it in nature, while in experiment the answer to a question is obtained by actively controlling and arranging the circumstances at will. There are no doubt some grounds for this distinction, though it is not true that the mind is passive in the one case and active in the other. Even in observation, as we have seen, knowledge always arises through active analysis and comparison of the instances selected as having a bearing on some problem. The difference is rather this: In observing, where experiment is impossible, one must wait for events

to occur, and must take them in the form in which they are presented in the natural order of events. But where experiment is employed we have control of the conditions and can produce the phenomena to be investigated in any order and as often as we choose. In experiment, as Bacon says, we can put definite questions to nature, and compel her to answer. This is of course an immense advantage. In some of the sciences, however — geology and astronomy, for example — it is not possible directly to control the conditions: one must wait and observe the results of nature's experiments. Physics and chemistry are the experimental sciences *par excellence;* and in general we may say that a science always makes more rapid progress when it is found possible to call experiment to the aid of observation. It is not possible to conceive how physics and chemistry could have reached their present state of perfection without the assistance of experiment. And the rapid advances made in recent years by biology and psychology have come mainly through the introduction of experimental methods. Indeed the comparative neglect of experiment by the Greek and mediæval scholars must be regarded as one of the chief reasons why the physical sciences made so little progress during earlier centuries.

We have seen that the distinction between observation and explanation is not an absolute one. The task thought has to perform — the task undertaken by science — is to reduce the isolated and chaotic experiences of ordinary life to order and system. And it is important to remember that all the various methods employed contribute

directly towards this result. It has however seemed possible to divide Inductive methods into two main divisions. Observation, it was said, seeks to discover the exact nature of the facts to be dealt with, and to find accurate means of describing and representing their qualitative and quantitative aspects. But when this has been accomplished we have not by any means reached an end of the matter. The desire for knowledge is not satisfied with a mere statement of facts, or even with a mathematical representation of them in a formula or a curve. Complete knowledge demands an *explanation* of the facts as determined by the methods of observation. The scientist is not content to know merely *that* such and such phenomena happen in certain definite ways, but he attempts to discover *why* this is so. 'Why', we ask, 'should dew be deposited at certain times, or water rise thirty-two feet in a pump'? The demand is that the processes of analysis be pushed farther by thought. What is required is a wider generalization, or the discovery of a more general law of behavior under which the phenomenon we are studying may fall as a special case. Yet this explanation, when arrived at, is on one side nothing more than a more complete description of the facts, calling attention to forces and happenings that escape ordinary observation. The explanation of the pump, for example, called attention to the weight of the atmosphere, hitherto neglected. But the new inductive step consists in something more than the addition of new facts. What is essential in explanation is rather the new way of colligating or thinking the facts in relation to one

another, afforded by the law or conception. The difference between Description and Explanation is obviously one of degree, being simply a question of how far analysis is pushed. In general we speak of a conception as explanatory rather than descriptive when it explicitly brings different facts into relation. Of course Explanation itself has various degrees of completeness and ultimateness. There always exists the ideal of a higher generalization, a more complete colligation of facts than any which science and philosophy have yet been able to achieve.

An excellent illustration of the distinction between descriptive and explanatory conceptions is afforded by a comparison of the work of Kepler with that of Newton. Kepler was filled with the idea that there must be some relation capable of mathematical expression between the different positions, previously determined by observation, in the orbit of the planet Mars. At length, after trying and discarding numerous other hypotheses, he was able to show that an ellipse could be passed through all these points. The proof was afterwards worked out of the elliptical character of the orbits of the other planets. The conception of an ellipse enabled Kepler to think all the observed positions of the planets in relation to one another. But the explanation of why the planets moved through elliptical orbits was still lacking. That explanation, as is well known, was given by Newton in his conception of universal gravitation. This was explanatory because it linked together the movements of the planets with the behavior of all other bodies moving in space, thus enabling the former to

be thought as examples or instances of the action of a universal principle.

It is usually said that where we know merely the nature of phenomena, and their connection, without being able to explain these facts, our knowledge is *empirical*. Thus I may know that an explosion follows the contact of a lighted match with gunpowder, or that a storm follows when there is a circle around the moon, without being able to explain in any way why these facts are connected. On the other hand if we can connect events by showing the general principle involved, we say that our knowledge is really scientific. It is important to notice, however, that empirical knowledge is simply in a less advanced stage than the scientific knowledge which has succeeded in gaining an insight into the general law; and also that any knowledge might be called empirical when contrasted with a more complete explanation. Thus Kepler's knowledge that the orbits of the planets are ellipses was empirical compared with that of Newton. Empirical knowledge leaves a problem which intelligence has still to solve. It is of course true that a large part of every one's knowledge is empirical in character. We all know many things which we cannot explain. In all the sciences, too, phenomena are met with which seem to defy all attempts at explanation. Indeed some of the sciences can scarcely be said to have passed the empirical stage. The science of medicine, for example, has hardly yet reached any knowledge of general principles. The physician knows, that is, as a result of actual experiment, that such and such drugs produce such

and such effects. But he knows almost nothing of the means by which this result is achieved, and is therefore unable to go beyond the fact itself. In this respect he is very little better off than the ordinary man, who knows that if he eats certain kinds of food he will be ill, or if he drinks strong liquors in excess he will become intoxicated.

EXERCISES (XIV)

1. What is the general assumption of all inductive thinking? Explain the relation of this assumption to the laws of thought.
2. What is the objection to considering resemblance as the basic principle of inductive inference?
3. What is the Uniformity of Nature? Is its existence consistent with (*a*) catastrophic changes, (*b*) miracle, (*c*) magic?
4. Distinguish between a fact, a fancy, and a theory. Give examples of each.
5. (*a*) In what respects does scientific observation differ from ordinary perception? (*b*) How does experiment differ from observation?
6. Experimentation always involves a purposive interference with phenomena. Explain. How can this fact be reconciled with the scientific ideal of 'objectivity'?
7. Explain, and illustrate by original examples, the difference between empirical and scientific knowledge.

CHAPTER XV

ENUMERATION AND STATISTICS

§ 58. **Enumeration or Simple Counting.** — We shall begin the account of the inductive methods with Enumeration. To count the objects which we observe, and to distinguish and number their parts, is one of the first and most essential operations of thought. It is of course true that qualitative distinctions generally precede quantitative. The child learns to distinguish things by some qualitative mark, such as 'black' or 'hot', before he is able to count them (cf. § 92). We may say, however, that the qualities of things are known, in a general way at least, before scientific procedure begins. The determination of quantity, on the other hand, seems to demand a more conscious effort on the part of the mind. We learn to distinguish the general qualities of things without effort; but to obtain exact quantitative knowledge it is necessary to set ourselves deliberately to work. And it is also necessary, as we shall see, to decide what we shall count. We must make up our mind, with some general idea more or less consciously before us, what it is worth while to enumerate. We may accordingly take **Enumeration,** or **Simple Counting,** which is perhaps the easiest kind of quantitative determination, as our starting-point in dealing with the Inductive Methods.

A considerable step in advance, in the task of reducing

the world of our experience to order and unity, is taken when we begin to count, *i.e.*, to group together things of the same kind, and to register their number. Thus Enumeration is to some extent also a process of classification. What is counted is always a collective whole, the units of which are either all of the same kind, or else belong to a limited number of different classes. Thus one might determine by Enumeration the number of sheep in a flock, taking each individual as belonging to the same general class, 'sheep'; or the analysis might be pushed farther so as to give as a result the number of white and of black sheep separately. The purpose for which the enumeration is undertaken always determines the length to which the process of analysis and distinction is carried. For example, if the object of a census enumeration were simply to determine the number of inhabitants in a country, it would not be necessary to make any distinctions, but each person would count as one. But where, as is often the case, the aim is not simply to count the sum-total but also to determine the relative numbers belonging to various classes, analysis has to be pushed further. In such cases we might count the number belonging to each sex, the native-born, and those of foreign birth, those below, and those above any given age, etc.

In Chapter XIII we have seen that the so-called 'Perfect Induction', where all instances are examined, is not properly called Induction at all, since there is no inference to anything new. Scientific Induction analyzes, notes special accompanying circumstances, and gets beneath the surface to the real or essential happening in the various cases. But

we saw that before the process of analysis is carried out, as well as in cases where the conditions are too complex or difficult to determine, we do proceed to generalize with greater or less confidence on the basis of the instances observed. If instances of P and Q, for example, have always been found in conjunction, and if we are confident that there has been nothing limiting or restricting observation to some special type of instance, we assume that the connection is not a mere 'casual coincidence', but that in some form it holds universally. In such cases the number of instances — *provided they can be assumed to be really unrestricted* — does seem to have a bearing on the logical character of the conclusion. The connection P — Q is less likely to be merely 'casual' in proportion to the frequency with which free, or unrestricted cases of it are observed, while at the same time no exceptions to it appear. The 'imperfect' character of the Induction, when based on a number of carefully established instances that show no exception throughout a considerable range, is found rather in the fact that the *nature* of the connection P — Q is left vague and undetermined, than in any lack of certainty regarding the existence of some universal principle of relationship. The frequent conjunction of a number of 'free' instances rules out the assumption of 'chance'; but in so far as the instances are left unanalyzed the precise form of the universal mode of connection is not exhibited in and through them.

Where experience shows both positive and negative cases, and where at the same time it is impossible to discover any

basis of difference for the two sets of results, we can compare the number of instances in which the connection obtains with that in which it fails. The ratio thus obtained may then be made the basis for calculating the probability of any particular event; or even of determining the likelihood that there is some law operative with regard to the observed phenomena (cf. p. 275).

As a matter of fact, however, Enumeration of instances is an aid to Induction mainly because in actual counting classification and analysis are also being effected. We are never content merely to count, taking each barely as 'one instance'. We also take account of the character of the instances, rejecting those that are not 'fair' or 'typical' and emphasizing others as of special or 'prerogative' importance. Moreover, the assemblage of instances of different types — of connection and lack of connection, of different races, or ages, etc. — serves to bring out differences and similarities between groups. In other words, statistics, when collected intelligently and with some problem in view, are really instruments of analysis; and in fields where experimentation is not possible they may be capable of revealing, not merely the fact that certain groups of things are correlated, but also to some extent the character of that correlation.

The conclusion we have reached is that no process of enumeration has any claim to the title of Perfect Induction. Enumeration is the beginning rather than the end of the inductive procedure. Nevertheless it is exceedingly useful as a preliminary step and preparation for scientific explana-

tion. The number of stamens and pistils a plant contains, or the number of tympanic bones possessed by an animal, is often of the greatest service in classification. And classification, although it is by no means the end of scientific investigation, is in many of the sciences a most essential and important step towards that end. The task of explaining the infinite variety of natural objects would be a hopeless one if it were not possible to discover similarities of structure in virtue of which things can be grouped together in classes. To this, enumeration in a very great degree contributes, especially if the counting is accompanied and directed by methodical thinking, so that the likenesses and characteristics enumerated are not taken at haphazard, but are really important ones, and such as to bring out, by means of the classification, answers to definite questions. Enumeration thus not merely groups together the phenomena to be studied in a compact form, but at the same time begins the process of analysis, revealing resemblances and differences.

§ 59. **Statistics and Statistical Methods.** — Statistical methods depend upon enumeration. They aim at making the process of counting as exact and precise as possible. Rümelin defines statistics as "the results obtained in any field of reality by methods of counting." Modern science has come to understand that its first task must be to become acquainted, as completely as possible, with the nature of the facts presented to it by experience. And for this purpose the careful classification and precise enumeration of particulars afforded by statistics is often of the greatest

importance. "The extent to which the statistical method prevails, and everything is counted," says Sigwart, "is another instance of the fundamental difference between ancient and modern science." [1] It would of course be impossible to enter here into a full description of the methods employed by statistical science. The methodology of every science must be learned by actual practice within the particular field. What we are interested in from a logical point of view is the purpose that statistical investigation seeks to fulfil, and the part it plays in rendering our knowledge exact and systematic.

We notice in the first place that the class of facts to which statistics are applied has two main characteristics: the subject dealt with is always complex, and capable of division into a number of individual parts or units; and secondly, it is also of such a nature that the underlying law or principle of the phenomena to be investigated cannot be directly discovered. Thus we employ statistics to determine the death-rate of any country or community, or the ratio between the number of male and of female births. It is clear that it is impossible to make use of experiment when we are dealing with facts of this kind because the conditions are not under our control. If it were possible, for example, to determine exhaustively the general laws according to which the various meteorological changes are coördinated with their conditions, we should not trouble ourselves to count and register the separate instances of changes in the weather. Nor if we knew exactly the general conditions under which

[1] *Logic* (Eng. trans.), Vol. I, p. 286.

any given human organism in contact with its environment would cease to exist, should we count the individual cases of death.

> In proportion as we are unable to reduce the particular event to rules and laws, the numeration of particular objects becomes the only means of obtaining comprehensive propositions about that which is, for our knowledge, fortuitous; as soon as the laws are found, statistical numeration ceases to be of interest. There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur, and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunder-storms and hail-storms occur at a given place, or within a given district, how many persons die, and how many bushels of fruit a given area produces, because we are not in a position to calculate these events from their conditions.[1]

In cases like those mentioned above, where we are as yet unable to determine the general laws at work, we call to our aid statistical enumeration. There are three main advantages to be derived from the employment of this method. (1) The use of statistics contributes directly towards a clear and comprehensive grasp of the facts. Instead of the vague impression derived from ordinary observation, statistics enable us to state definitely the proportion of fine and rainy days during the year. Statistical enumeration is thus one of the most important means of rendering

[1] Sigwart, *Logic* (Eng. trans.), Vol. II, p. 483.

observation exact and trustworthy, and of summing up its results in a convenient and readily intelligible form. It is of the utmost importance when dealing with complex groups of phenomena to have a clear and comprehensive view of the facts of the case. Thus when trying to understand the nature of society it is necessary to determine accurately, by means of statistics, such facts as the number of male and of female births, the death-rate, the proportion of marriages, the age of marriage, etc. This may be regarded as the descriptive use of statistics. (2) In the second place, by giving us the average in the past for large numbers of things or events occurring within certain lengths of time, in areas of space, statistics enable us to form probable judgments as to what will happen in the future in cases where we cannot predict because the causal laws are unknown or are too complex. This second use will be further discussed in § 60. And finally, (3) statistics often serve to reveal quantitative correspondences or uniformities between two groups of phenomena, and thus suggest that some causal connection exists between them. It is found for example that the number of births in any given country tends to vary in relation to the abundance or scarcity of food. Now this fact at once suggests the existence of laws which will serve to bring these facts into causal relation. In many cases such correspondences serve only to confirm our expectation of the presence of a causal law, which is based on other grounds. Thus we should naturally expect that there would be a relatively greater number of cases of fever in a town having an insufficient water supply, or an antiquated system of sew-

erage, than in a town where these matters were properly provided for; and statistics might bear out our conclusions. In general, however, it may be said that causal laws are suggested, not by corresponding uniformities, but by corresponding variations, as shown by the statistics of different sets of facts. So long as the death-rate shows a constant ratio to the population, no causal inference is suggested; but if the annual number of deaths increases or decreases considerably we are led to look for some variation from the normal in some coincident group of phenomena. And if it is found that the variation in the death-rate has been accompanied by unusually favorable or unfavorable conditions of weather, the presence or absence of epidemics, or any similar circumstances, there will be at least a *presumption* that a causal relation exists between these two sets of events. From a certain likeness or quantitative proportion between the variations of two distinct classes of phenomena, we are led to the hypothesis of their causal connection.

In this use of statistics, they become directly auxiliary to an explanation of the facts they enumerate. But the correlation and causal connection of the facts come to light only when looked for. Merely to count, without any definite purpose, would never help us to explain. As we saw in the last chapter, induction always proceeds under the guidance of conceptions or general ideas. We do not simply stare, as it were, at the facts we examine, but we look at them to discover their meaning and select such of them as are relevant or significant in the light of some general theory or conception. In other words we examine the facts to put

theories (which may of course be very vague as yet) to the test, or to get answers to certain questions which we have in mind. Now this is just as true of enumeration and statistics as it is of the other methods of induction. As has already been remarked of enumerative classification, we must decide what it is worth while to count in the particular field in which we are counting. The questions that we wish answered will determine this. And even when we have our figures, they will be meaningless or even altogether misleading unless we know how to interpret them. It is the neglect of such considerations that leads to the misuse of statistics and the frequent contradiction of the statement that 'figures cannot lie'.

It is true that on a superficial view of the statistical method the figures may seem at times to arrange themselves in definite groups quite apart from any intellectual labor save that of mere counting. Thus it might seem that in taking the average rate of mortality on the basis of the returns of local officials, etc., the figures of themselves disclosed the fact that the rate was higher for infants under two years of age than in later periods of life. But the total average of deaths would never have shown this. It is only because the average for infants has been separately calculated, *in the expectation that there might be a difference*, that the difference has been found. The tentative question — Is there, as we have reason on the ground of unsystematic observation to believe, a striking difference between the death-rate of infants and that of older persons? — is thus answered in the affirmative.

But the function of guiding ideas and hypotheses becomes even more important when the statistics are to be used directly in the service of explanation. Two examples will serve to make this plain. The first is from Sigwart:

> The position of a barometer in a given locality passes from day to day, and from month to month, up and down through all possible variations, in which we can at first find absolutely no rule (though they have a constant mean value). . . . But if we calculate the average for the particular hours of the day over a considerable time, we find a periodical variation between two maxima and minima with respect to the general average. . . . That the period is daily points to the influence of the sun. . . . But unless we had conjectured that the different positions of the sun, and the changes brought about by them, had some influence, we could not have thought of summing up the particular hours of the day apart from each other.[1]

In this case the constant average first obtained told us nothing, except that the conditions, whatever they were, which governed the fluctuations of the barometer, remained constant on the whole. But when an hypothesis was found, and the varying positions divided into groups of such a nature that their comparison could test it, we obtained a partial explanation of them.

Again, suppose that we are gathering statistics of the divorce-rate in various states and countries. The figures, unanalyzed, would tell us little. But suppose we had a definite problem in mind, such as the effect of laws on the

[1] *Logic* (Eng. trans.), Vol. II, pp. 496–497.

frequency of divorce. What would we do with our figures? "First, select states or countries with similar social and economic conditions, but very different laws, and compare their divorce-rate; do the same for states with similar laws, but different economic conditions; note whether the divorce-rate varies with the law, or with the other factors, or with neither exclusively. Secondly, examine every instance of a change in the divorce law, and observe whether it was attended by a change in the figures such as might have been produced by the law." [1] Here again there is a division of the phenomena into groups distinguished by some difference in the supposed cause, and then a comparison of these groups. The methods employed, as we shall see presently, are essentially those of Agreement and Difference, and of Concomitant Variations.

In general, then, there are two things to be said about the use of statistics. In the first place, the smaller and more numerous the groups are into which the enumerated phenomena are divided, and the more exactly the rules of division in general are followed in doing this, the more valuable, other things being equal, the statistics will be. In the second place, it is by the comparison of these groups that statistics aid us to discover causal or other relations. The kind of groups we shall make, and the points in which we shall compare them, are determined by the questions we have to ask, or the tentative conceptions we have to test. In all these respects the use of statistics is governed by the general principles of the inductive method, which con-

[1] Willcox, *The Divorce Problem*, p. 41.

sists essentially in the analysis and comparison of phenomena in the light of an hypothesis.

Statistical enumeration is frequently employed to determine the *average* of a large number of instances of a particular kind. This is obtained by dividing the sum of the given numbers by the number of individuals of which account is taken. In this way a *general average* is reached which does not necessarily correspond exactly with the character of any individual of the group. It represents a purely imaginary conception, which omits individual differences and presents in an abbreviated form the general character of a whole class or group. In this way, by the determination of the average, it becomes easier to compare complex groups with one another. Thus when the average height of Frenchmen and Englishmen has been determined, comparison is at once made possible. From the mean or average of a number of individuals, or set of instances, however, we can infer nothing regarding the character of any particular individual, or of any particular instance. What *is* determined by the method of averages is the general nature of the group, as represented by the average or typical individual. But this method does not enable us to infer anything regarding the character of any member of the group, A, or B.

Indeed the simple arithmetical mean or average by itself may give us quite an erroneous idea of the general character of the individuals or instances making up the group. For example, if ten divorces were granted in a county, eight at the end of three years of married life, one at the end of six, and one at the end of thirty, it would give quite a misleading

notion to say that the average duration of marriage in couples seeking divorce there was six years. In order to correct such defects in the use of the average by itself, especially in applying the statistical method in biology, two other expressions are now used, the *mode* and the *median value*. The *mode* is the condition which occurs most often in the group examined; in the example just cited it would be three years. The *median value* is the condition of the individual at the middle of the series, when it is arranged in order. In this case it approximates to the mean. When the group is symmetrically distributed about the average, these three expressions are approximately the same; but as it becomes less evenly distributed, they differ more or less widely, and now one of them, now the other, may give a better notion of the character of the group than the average by itself would. All three expressions, however, are primarily expressions for the general nature of the group; and the information they give us concerning the nature of any individual member of it is always indirect, imperfect, and uncertain, save as we are informed where in the group the member occurs. There are also occasions when it is preferable to use the *geometrical mean*.

And finally there is the *weighted* average, *i.e.*, one whose constituents have been multiplied by certain weights before being added, the sum thus obtained being divided by the sum of the weights instead of by the number of items. The 'index numbers' of the economist and of the financial expert constitute one important type of these averages, while another is illustrated by the following example. "If a

department store had seven departments and the average wage for each department was known, to obtain the weighted average it would be necessary to multiply the average wage for each department by the number of workers in it, add the results together, and divide by the total number of workers."[1]

§ 60. **The Calculation of Chances.** — We still have to consider the second of the three uses of statistics mentioned in the foregoing section. As has been said, statistics not only help us in describing and in explaining complex phenomena, but they are also used to enable us to judge what will be true, on the whole, of a long series of events, in cases where ignorance of the causal laws concerned prevents our making predictions concerning the individual members of the series, when taken separately. This is usually called the calculation of chances, or probabilities. In the physical sciences what are known as 'statistical laws' have an important place, and in the extremely difficult domain of quantum mechanics the calculation of probabilities seems to be all that is possible at the present time. But contrary to a rather wide-spread misconception, this state of affairs does not support the view that there is such a thing as 'chance', regarded as a power which controls and governs events. When we speak of something happening 'by chance', or of some occurrence as 'probable', we are expressing merely a deficiency in our own knowledge. "There is no doubt in lightning as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach but infinite

[1] Sellars, *The Essentials of Logic*, p. 249.

knowledge would account for its lying there; and the course of every falling leaf is guided by the same principles of mechanics as rule the motions of the heavenly bodies."[1] To assert that anything happens by chance, then, is simply to confess our ignorance of the causes that are operative.

It is clear that we are in this position regarding many of the ordinary events which belong to the future. Because of my ignorance of the causes at work, I can only say, 'It *may* rain to-morrow'. It is impossible to tell upon which side a penny will fall at any particular throw, or what card may be drawn from a pack. But in cases like these, we have to accept, for lack of anything better, a numerical statement of the chances for any particular event. Thus we know that, since there are only two sides upon which a penny can fall, the chances of throwing heads in any trial is $\frac{1}{2}$. Similarly there are four chances out of fifty-two of drawing an ace from a pack of cards. The chance of obtaining an ace by any draw is therefore $\frac{4}{52} = \frac{1}{13}$. These figures express the mathematical chances. Experience of a limited number of instances may however sometimes appear to show a lack of harmony between the mathematical and the actual chances. But in proportion as the number of trials is increased the result is found to approximate more and more nearly to the mathematical expectation. In twenty throws of a penny or a die we should not be surprised to find that the result differed from the fraction expressing the mathematical chances. But this discrepancy would tend to disappear as the number of cases was increased. Jevons illus-

[1] Jevons, *The Principles of Science*, Vol. I, p. 225.

trated this by actual trial, using a number of coins at a time. Out of a total of 20,480 throws, he obtained a result of 10,353 heads. On the result of the experiment he remarks: "The coincidence with theory is pretty close, but considering the large number of throws there is some reason to suspect a tendency in favor of heads." [1]

Apart from the simple and somewhat artificial cases where we are concerned with coins and dice, etc., it is impossible to determine with mathematical precision the chances for or against any event, since the possibilities are indefinite as well as the causes. In cases where the whole series of possibilities does not lie before us we have to base our calculations for the future on what is known regarding the frequency with which the events under consideration have occurred in the past. Now the results of the last paragraph make it clear that it is of the utmost importance that the statistics which are taken as the basis shall be as full and comprehensive as possible. It is evident, for example, that serious errors would be likely to arise, if the death-rate for a single year, or for a single county or town, were taken as typical of the country as a whole. To render statistics trustworthy they must be extended over a considerable period of time, and over a large extent of country, so as to eliminate the accidents due to a particular time or to a particular locality.

When this has been done, however, and statistics have been obtained that have a right to be regarded as really typical, the chances in any individual instance regarded

[1] Jevons, *op. cit.*, Vol. I, p. 230.

simply as one member of a large group, and apart from its own special characteristics, can be readily shown. Thus we find that out of one thousand children born, about two hundred and fifty die before the age of six years. The chances, then, at birth, that any child will reach this age, are $\frac{750}{1000}$ or $\frac{3}{4}$. Again, it is found that only about two persons in one thousand live to be ninety years old. So that the probability of any child living to this age would be expressed by the fraction $\frac{2}{1000}$ or $\frac{1}{500}$. Such probabilities are simply averages which briefly describe what has happened in the past. Now what has happened in the past in a large number of cases we naturally expect to happen in the future. This is essentially the principle upon which life-insurance companies proceed. Their business is conducted on the assumption that there will be an approximately constant death-rate, though they cannot foretell what particular individuals are to die in any year. It thus becomes possible to calculate what losses from death may be expected each year. Suppose that it is found that the annual death-rate among men of a certain age throughout the country is twenty out of every thousand. If each man's life were insured for $1000, the loss to the company from this source would be $20,000. To compensate for this loss the company would be obliged to demand an annual payment of $20 from each of the one thousand individuals in the class. Of course the actual computations upon which insurance is based in concrete cases are vastly more complex than this, and many other considerations arise of which account has to be taken. But the general

principle involved is, that by taking a sufficiently large number of cases, chance can be almost eliminated. We can have no means of determining whether any healthy individual will or will not die before the end of the year. There would be a very serious risk, amounting practically to gambling, in insuring his life alone, for probabilities are essentially averages. They inform us about the group, and not directly about any particular member of it. But the transaction, as we have seen, is no longer a mere speculation when a large number of individuals are concerned; for the actual loss can be accurately foretold and provided for.

As precise an analysis of the conditions as is possible is as important in estimating probabilities as it is in the other uses of statistics. The smaller the group of which the average is taken, and the more definite the information we have about it, the more accurate our estimate becomes. It is not enough, for example, for the purposes of life-insurance, to know what the average age of death is, all adults being taken as on the same footing. What the insurance companies do is, in the first place, to exclude all who are not in fairly good health, and who may be in danger of hereditary disease, from their membership; and, in the second place, to calculate the average number of years of life remaining to men of different ages. Every individual is thus put into a special class and the premium calculated accordingly.

A rather common fallacy is to suppose that the known probability of any particular event of a group or series

gives us some ground for expecting this when the other events of the series have occurred. But it should be remembered that the known probability affords no such ground of inference except as we know that there is some causal relation between these events; and then we are not reasoning by probabilities. The probability of throwing double six with two dice, for example, is $\frac{1}{36}$. But because in thirty-five consecutive throws the double six has not appeared, it does not follow that it is any more likely to do so on the thirty-sixth throw than it was on the first. The probability is still $\frac{1}{36}$, and so continues. If we take a sufficiently large number of throws, as has already been remarked, we shall find that the double six has, *on the average*, appeared once out of every thirty-six throws. But we cannot foresee whether the appearances of the double six sufficient to give this average will be evenly distributed through the whole series of throws, or occur in irregular sequences.

A peculiar use of the theory of probability in order to discover causal connections between events is possible on the principle just stated. When we are in doubt, that is, as to whether two events are in any way causally connected, we can by collecting statistics estimate the probability of their appearing together on the assumption that they have no causal relation. Then if they are found to appear together more or less frequently than this estimate, we are justified in assuming that there is some causal relation between them. Suppose, for example, we are studying two characteristics which occasionally appear in a certain species of animal, and wish to determine whether they have any

essential connection. We find on examining a large number of cases that one of these characteristics appears once in every sixteen individuals, on the average, and the other once in every twenty. If there is no connection between them, then, on the theory of probability, the chance of their happening *together* is $\frac{1}{320}$. But if we found that they occurred together in twenty cases out of every hundred, we should conclude that there must be some cause or causes common to both characteristics, or else that one of them in some way depends on the other.

EXERCISES (XV)

1. What is the justification for beginning our account of the inductive methods with Enumeration?
2. What influence does the fact that a large number of instances have been examined have on the validity of an inductive inference?
3. State and distinguish three uses to which statistics may be put.
4. What is the relation between the warning that observation and inference should not be confused, and the frequent claim that statistics are misleading?
5. How may statistics be used to prove or disprove a law of causal connection? Illustrate your answer.
6. In what respects do statistical conclusions differ from empirical generalizations?
7. Illustrate by original examples the meaning of the terms, average, mode, median, geometrical mean, and weighted average.
8. Discuss the conception of 'objective chance'. Is there any sense in which the physical world is contingent?

CHAPTER XVI

DETERMINATION OF CAUSAL RELATIONS

§ 61. **Causal Connection.** — So far we have been dealing, primarily, with observational methods, and with the results obtained through the enumeration of particular things. We have been considering how our knowledge of the qualities and quantities of objects may be made as exact and complete as possible, but we have not discussed in detail the methods by which we discover the connection of things. But all inductive thinking, as has been shown, is based on the assumption that there are universal forms or principles of relation according to which things are connected in a systematic way. We cannot really be said to know at all, until we become aware that certain parts of our experience are united, like the links of a chain, one part involving another. And as has been already frequently pointed out, the growth of knowledge is constantly bringing to light new connections between facts that were previously taken to be independent of one another. The physical sciences, however, in describing and explaining the relations of things, do so primarily in terms of Cause and Effect. All phenomena without exception, it is assumed, are causally dependent on other phenomena; everything that happens has its cause, and is in turn followed by its effect. From the standpoint of practical experience, also, we are con-

stantly obliged to look for causes; for only where the cause is known is there any certain method of producing the effect. The determination of causes, then, is one of the most essential problems of Induction, the category of Cause and Effect being perhaps the most universal and important category by means of which the parts of our experience are thought as related according to universal laws. What rule, or rules, can now be given that will enable one to discover what is the cause or the effect of an event in any particular case?

Before we proceed to the answer of this question, however, it is necessary to explain briefly what is meant in the physical sciences by the relation of cause and effect. In the first place these sciences regard the world as consisting of a phenomenal order of events. In other words, they are concerned with the particular things and changing events that *appear* or *show themselves* in ordinary experience. Both the inner and the outer world appear to be composed of an indefinite manifoldness of particular things, events, occurrences. Now, the physical sciences do not ask whether this aspect of the world is ultimate Reality or merely Appearance. The problem of the scientist is rather to set out from the manifold objects and events as they appear in ordinary experience, and to seek to describe and explain them by showing how they are related in various complex ways through principles of causal dependence. It is assumed that each phenomenon of which the world is composed, is yet, in spite of the independent and separate existence it seems to have, connected through the principle of causality

with something else which determines it, or is in some way necessary to its existence. Every event, that is, has its cause. The explanation of every phenomenon is to be found in something external to it, but upon which it is dependent. The relation of cause and effect assumes that all phenomena are *externally* determined; or, as the same thing is often expressed, it assumes a *mechanical* relation between the different parts of the world. Moreover, this relation is simply a special form, or category, through which the universal relations of things are expressed. That there are universal modes of connection, and that 'once true always true', is a law or postulate of all thinking. Causality, being as we have seen one very definite and useful way of thinking that relation, is accordingly of the greatest importance, both for science and practical life.

When the general postulate of all thinking, that things shall hold together systematically so as to be intelligible, is put in more definite form as the law of Cause and Effect between phenomena, we get the notion of the Uniformity of Nature. Strictly speaking, the Uniformity of Nature is involved in the fundamental postulate of thought that things hang together in a rational way. Nevertheless the conception is usually taken to imply the absolutely invariable sequence of causal events. From the point of view of natural science, Nature is uniform in the sense that all instances of the same phenomenon P are always determined in the same way by the same cause Q. This, then, is really mechanical uniformity. The relation between P and Q is

not only external or mechanical, but absolutely fixed and invariable. The conception of any 'spontaneous variation', any modification without an externally determining cause, is completely excluded.

In speaking of any phenomenon as having *a* cause, the relation has of course been artificially simplified. In reality there are always a number of 'causes', or determining conditions necessary to the occurrence of any event. What we mean by 'the cause', in any particular case, depends mainly on the character and purpose of the inquiry. In practical life the 'cause' sought for is usually something that can be employed directly as a means to the desired result. And even in scientific inquiries practical motives continue to play a part in deciding what shall be regarded as the 'essential' or 'real' cause of any phenomenon. The cause is whatever can be employed to produce the desired effect, and so to afford practical mastery over the situation. This direct reference to practice, however, is not essential to the idea, which is primarily a way of thinking things in relation. Ultimately, the 'real' or 'essential' cause is that which shows most clearly the character of the relationship between two phenomena — that which, in a sense, is the sum or synthesis of all the conditions.

The cause, then, from the point of view of science, is that without which the phenomenon would not occur. It is also sometimes defined as 'the invariable and necessary antecedent', while the effect is spoken of as the 'invariable consequent'. In using these terms, however, it must not be supposed that the cause always and necessarily precedes

the effect in time. The relation of cause and effect is not to be regarded as merely temporal.

§ 62. **Mill's Experimental Methods.** — The methods by which causes and effects may be determined were formulated by Mill in his *Logic*. He stated, in general terms, the principles already in use in scientific procedure. However, both Bacon in his *Novum Organum* and Hume in his *Treatise on Human Understanding* anticipated Mill in this attempt to formulate certain methods or rules designed to do for induction what the rules of syllogistic reasoning supposedly had accomplished for deduction; namely, to reduce the process of inference to the quasi-mechanical application of set rules to given data or premises. But we have already seen that only the simplest types of deductive inference can be arranged in the form of categorical syllogisms, and our exposition and analysis of Mill's methods will show that they too are of much less significance for inductive inference than has sometimes, uncritically, been supposed.

Mill gives five separate canons, but as he himself recognizes, there are but two main principles involved. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. One is, by comparing together different instances in which the phenomenon occurs. The other is, by comparing instances in which the phenomenon does occur with instances in other respects similar in which it does not. These two methods may be respectively denominated the

Method of Agreement and the Method of Difference." [1] Of the other three methods mentioned by Mill, one — the Joint Method of Agreement and Difference — is, as the name implies, a direct combination of the first two, while the Method of Residues and the Method of Concomitant Variations are corollaries from the same principles.

The purpose of these comparisons is to *exhibit and define the true cause.* This is accomplished by proceeding directly through negation. That is, the other circumstances which could be supposed to have any influence are successively eliminated. And as already pointed out (§ 54), it is just with a view to the possibility of elimination that the instances are selected. Since the cause is that without which the phenomenon would not occur, the rules of elimination follow immediately: (1) That is not the cause of a phenomenon in the absence of which the phenomenon occurs; (2) That is not the cause of a phenomenon in whose presence the phenomenon fails to occur; (3) That is not the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it. [2]

The process of eliminating the other things that could conceivably be causes also defines the sphere and nature of the true cause. The preceding rules, then, might have been stated positively, and it is this positive side of the

[1] Mill, *Logic*, Bk. III, Ch. VIII, § 1.

[2] These statements are essentially those given by Joseph (*An Introduction to Logic*, 2nd ed., p. 429), who however adds a fourth supplementary rule: "Nothing is the cause of one phenomenon which is known to be the cause of a different phenomenon."

process that Mill emphasized. It is important to bear in mind, however, in studying these *Methods of Experimental Inquiry*, that elimination or negation plays an important part in the process he describes. We shall now proceed to state and illustrate the canons of the different methods.

§ 63. **The Method of Agreement.** — The principle upon which this method proceeds is stated in the following way by Mill: "*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.*" The purpose of this rule, it will be remembered, is to help us to determine what particular facts in our experience are connected as causes and effects. If the problem is to find the cause of some phenomenon, the canon may be illustrated in the following way. Let P^1, P^2, P^3, represent different instances of a phenomenon, P, whose cause is to be ascertained. And suppose that we are able to analyze,

the antecedents of P^1 into *abcd;*
the antecedents of P^2 into *gfcm;*
the antecedents of P^3 into *klnc.*

Now it is clear that *c* is the sole circumstance in which the antecedents of all these instances of P agree. And nothing can be the cause of P in the absence of which P still occurs. We should be justified in concluding, therefore, according to this method, that *c* is *probably* the cause of the phenomenon under investigation, P.

If now we wished to discover the effect of something

which happens, it would be necessary to determine, by observing a number of instances, what common circumstance can be found among the events which follow it.

If Q^1 were followed by *fghk*,
and Q^2 were followed by *lmgc*,
and Q^3 were followed by *grst*,

we should be able to say that Q and *g* were *probably* connected as cause and effect.

When antecedents and consequents are thus represented schematically by means of letters, it is easy to perceive at once the common circumstance in a number of instances. But the facts and events of the real world are not separated off from each other in this way. The common circumstance in which a number of instances agree has to be separated out by analysis from the variable elements forming part of the different antecedents and consequents. Moreover, an essential part of the work of Induction consists in selecting instances such that all the possibilities — all the things that might be connected with P — are included. It should also enable us to recognize the common element as common, though it may appear in wholly different circumstances.

If a number of cases of typhoid fever were to appear at about the same time in a community, one would naturally wish to explain this phenomenon by tracing it to its cause; and to do this one would try to discover some circumstance which was the common antecedent of all the cases. Knowing from the records of past experience that the cause is to

be sought for among a limited number of circumstances, one would select the various instances with the purpose of testing the different possibilities. The water supply might first be examined. But if it were found that this was derived from entirely different sources in the different cases, we should probably conclude that the explanation must be sought elsewhere. Suppose that as a result of careful analysis it were discovered that all the individuals prostrated with the fever had eaten oysters bought at the same market. If this were the only common circumstance discoverable after careful investigation, we should conclude that *probably* the oysters were the cause of the fever. The process of analysis could be pushed still further, if one wished, in order to determine more exactly the precise source of the infection; *e.g.*, it might be found, as a result of further inquiry, that the water in which the oysters were kept was vitiated by a sewer.

It is important to note that the conclusions reached by this method are greatly strengthened by increasing the number of observations, and by taking as many instances as possible that are dissimilar in character. By so doing the real cause is more likely to be included among the antecedents noted, and at the same time the probability is lessened that the connection between antecedent and consequent is a merely accidental conjunction. But even when such precautions are taken the method of Agreement does not afford any very definite knowledge. By eliminating the other antecedents we found that *c* is probably connected causally with P. But *c* is left as a mere unanalyzed

'circumstance', *e.g.*, 'the drinking water', etc. Just how the connection takes place, and whether it be direct or indirect, is not shown. It is clear, then, that further analysis is necessary in the interest of scientific knowledge. The method of Agreement, although perhaps in some cases yielding results sufficiently exact for practical application, merely suggests a problem for further inquiry. Its defect, as we have seen, is that it does not sufficiently get beneath the surface of things so as to make certain and definite their mode of relation.

§ 64. **The Method of Difference.** — According to the method of Agreement we compare a number of diverse instances, in all of which a given phenomenon occurs, and endeavor to discover the one circumstance invariably present. The method of Difference, on the other hand, compares an instance in which a phenomenon occurs with another as nearly similar to it as possible, in which it does not occur. Its canon is expressed by Mill as follows: "*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" It will perhaps make the matter clearer to say: That which is present in a case when a phenomenon occurs, and absent in another case when that phenomenon does not occur, all other circumstances remaining the same in the two cases, is causally connected with that phenomenon. That is, by means of this method we compare two

instances differing only in the fact that the phenomenon in which we are interested is present in the one and absent in the other. If now the two cases are represented in this way,

PHK conjoined with *alg*,
and HK conjoined with *lg*,

we conclude at once that P is causally connected with *a*. Our selection of P, or the element in question, as the supposed cause, is of course made in accordance with an hypothesis or general notion of what the possible or likely causal relations in the subject under investigation are, gathered from previous experience. If this notion is as yet too vague to give us any definite guidance we are obliged to analyze the phenomena as exactly and minutely as we can, and experimentally vary the circumstances in every conceivable way, until the requirements of the method are, if possible, satisfied.

Almost any instance in which experiment is employed will serve to illustrate this method. If a bell is rung in a jar containing air the sound will of course be heard at any ordinary distance. But after having removed the air by means of an air-pump, let the bell be struck again. It will now be found that the sound is no longer heard. When the two cases are compared it is at once evident that the only difference in the antecedents is the presence of the air in the one case and its absence in the other. When the air was present the sound was heard; when it was absent the sound was not heard. We therefore conclude that the perception of sound is causally connected with the presence of atmospheric air.

Again, we can prove that the so-called 'taste' of different objects depends upon smell, by tasting, say, an orange, and after a little time has elapsed, tasting it a second time while holding the nose. It will be found in this latter case that instead of the familiar 'orange taste', one senses merely 'acid', or 'sweet'. The only difference in the two trials being that the organ of smell excluded in the latter, was operative in the former, it follows that the so-called 'orange taste' is proved to be due to smell rather than to taste proper.

An essential requirement of the method of Difference is that *only one circumstance shall be varied at a time.* The object of the method is to isolate the various conditions which go to make up a complex phenomenon, in order that we may mark the effect of the presence or absence of each one individually. Now in observing what goes on in nature we rarely find changes in which but a single element has varied. If we find that to-day is cooler than yesterday we may be inclined to refer the change to the thunder-storm of last night. But rain also accompanied the thunder-storm, and the direction of the wind has changed. So that it is impossible in such cases to apply the method of difference. To employ this method successfully observation usually must be supplemented by experiment. In performing experiments we determine what conditions are to be operative, and arrange the apparatus so as to carry out our purpose. Having thus control of the conditions we are able to vary them at pleasure. In this way experiment becomes an instrument by means of which analysis can be

carried further than is possible for unaided observation. It enables us to separate things usually conjoined, and to observe the result of each when taken by itself. In employing experiment, however, the greatest care must always be taken to introduce or remove only one condition at a time, or at least only one new circumstance which can in any way influence the result.

It often happens, too, as Jevons points out, that the experimenter is not aware of all the conditions operative when his investigations are made. "Some substance may be present, or some power may be in action which escapes the most vigilant examination. Not being aware of its existence, we are of course unable to take proper measures to exclude it, and thus determine the share which it may have in the results of our experiments."[1] For this reason it is always necessary that experiments should be repeated by different persons, and so far as possible under varying conditions. I quote an example from the work of Jevons to which reference has just been made.

> The great magnetic power of iron renders it a constant source of disturbance in all magnetic experiments. . . . In some cases, magnetic observations have been seriously disturbed by the existence of masses of iron in the neighborhood. In Faraday's experiments upon feebly magnetic or diamagnetic substances, he took the greatest precautions against the presence of any disturbing substance in the copper wire, wax, paper, and other articles used in suspending the test objects. It was his invariable custom to try the effect of the

[1] Jevons, *Principles of Science*, Vol. II, p. 37.

magnet upon the apparatus in the absence of the object of experiment, and without this preliminary trial no confidence could be placed in the results.[1]

It is sometimes impossible to remove the suspected cause experimentally without materially changing the attendant circumstances; or it may be impossible to remove it at all, as in the case of gravity. But this difficulty may often be overcome by introducing a circumstance which overcomes or neutralizes the effect of the supposed cause without altering the rest of the phenomena.

§ 65. **The Joint Method of Agreement and Difference.** — The method of Difference can be applied only when all concomitant circumstances, except one, remain constant. In order to apply this method, then, it is necessary either to find two instances differing only in a single circumstance, or to proceed by means of experiments, adding or removing a single circumstance at a time and noting the result. The difficulty is to find instances that differ only in a single circumstance in fields where, from the nature of the case, experiments cannot be used. For example, in trying to reach generalizations regarding the behavior of human individuals or human societies — in looking for moral, or social, or economic laws — it is of course impossible to employ experiment. Nor when dealing with individuals and societies can we find two instances which certainly differ from each other in only a single circumstance. In studying phenomena of this kind it is necessary to employ another method as an instrument of analysis. What is done by this

[1] Jevons, *op. cit.*, pp. 40, 41.

new method is to take a number of instances instead of only two. A number of instances where the phenomenon to be investigated occurs are compared together, and likewise a number of instances where it does not occur, and the results of the two comparisons noted.

This is really to combine the principle of the method of Agreement with that of the method of Difference. Mill accordingly has called this the Joint Method of Agreement and Difference, and has given the following statement of its canon: "*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*"

In interpreting this canon it is important to remember that both positive and negative instances must be selected from the field within which our previous knowledge enables us to say that the cause (or effect) sought for is to be found. The purpose of the instances, as has been frequently pointed out, is to bring to our attention circumstances which might conceivably make a difference. It is of course impossible to predict in advance all the things that might make a difference, but the possibilities fall within a more or less definite range. In both the positive and negative set of instances, then, we are concerned only with circumstances that might be relevant. The negative instances to be chosen are therefore not *any* cases 'where the phenomenon does not appear',

but where in addition circumstances previously found in conjunction with the phenomenon, which might have been supposed to be causally connected with it, are now shown to be sometimes, at least, present when it is absent. To represent the working of the matter schematically we may analyze the instances where the phenomenon P occurs into the following circumstances: —

Instance 1 . . . . *a, b, c, d, e.*
Instance 2 . . . . *f, c, a, g, k.*
Instance 3 . . . . *d, m, b, c, e.*
Instance 4 . . . . *k, n, c, g, a.*

The method of Agreement, in such a case, would lead to the conclusion that *c* is probably connected causally with P. To strengthen and render more definite that conclusion, however, the Joint method introduces the comparison of instances as much like the former group as possible and known to exhibit at least many of the same circumstances, but where the phenomenon in question does not occur. These instances of the absence of P would then be represented thus: —

Instance 1 . . . . *b, k, n, g, a.*
Instance 2 . . . . *d, e, b, m, f.*
Instance 3 . . . . *k, l, s, g, b.*
Instance 4 . . . . *x, e, n, a, f.*

What is of significance in this latter series is not merely that the instances show nothing common except the absence of P, but that the same 'circumstances' excluded by the former analysis are now seen to exist in the absence of that

phenomenon. But what may be present when a phenomenon is absent is not its cause or effect. All these possible circumstances, then, *a*, *b*, *d*, etc., are again eliminated by the comparison of negative instances, leaving as before *c* as that which is causally connected with P.

As an illustration of the method of Agreement and Difference the following instance will serve: —

We may suppose that in a certain part of the country it was noticed that a considerable difference existed in the number of criminal offences committed, in proportion to the number of inhabitants, in the various towns. In several towns the percentage was high while in others it was relatively small. This being so, a question naturally arose as to the cause of the high percentage. Now there were among the people various opinions concerning the matter. One thought it was due to the small number of police, a second believed it was caused by the inefficiency of the public schools, a third attributed it to the inadequacy of the penalties attached to the violation of law, a fourth was convinced that it was due to lack of activity on the part of the churches, while a fifth insisted that the phenomenon could be accounted for by the presence of licensed saloons. Not being able to agree about the matter, it was decided to appoint a committee to investigate the circumstances existing in various towns where the same general conditions prevailed, and upon the basis of this comparison to reach a conclusion. The towns with a high criminal percentage were examined first. The report of conditions there was as follows: —

Town A: Small police force — efficient schools — severe penalties — inactive churches — licensed saloons.

Town B: Small police force — efficient schools — light penalties — active churches — licensed saloons.

Town C: Large police force — inefficient schools — severe penalties — active churches — licensed saloons.

Town D: Large police force — inefficient schools — light penalties — inactive churches — licensed saloons.

This report revealed the fact that in each of these towns having a high criminal percentage there was one circumstance, and only one, invariably present — the licensed saloon. This rendered it probable that the saloon was the cause of the high precentage of crime. But before finally deciding it was thought well to investigate negative instances as well; that is, towns in which the high percentage of crime did not occur. The report of conditions there was as follows: —

Town E: Large police force — efficient schools — severe penalties — active churches — no licensed saloons.

Town F: Large police force — inefficient schools — light penalties — active churches — no licensed saloons.

Town G: Small police force — efficient schools — light penalties — inactive churches — no licensed saloons.

Town H: Small police force — inefficient schools — severe penalties — active churches — no licensed saloons.

This table showed that in the absence of the phenomenon (high criminal percentage) one and only one of the conditions concerned was invariably absent; namely, the licensed saloon. This confirmed the previous report and

established to the satisfaction of all that the saloon was, at least, the main cause of the high criminal percentage in the cities concerned.

Of course it is obvious that this can be no more than a hypothetical case. In actual life the conditions of the method would never be so exactly realized. In the first place, in any such investigation, it would probably never be possible to find instances where one condition is invariably present when the phenomenon occurs and invariably absent when it does not occur, as the illustration supposes. We could at most expect that one condition would exhibit a *tendency* to be present when the phenomenon occurs and absent when it does not occur. That is, there might well be instances met with in which a combination of other conditions might render unnecessary the presence of the usually essential one. In the second place it would not be satisfactory in actual life to deal with such vague terms as 'efficient' schools or 'active' churches. On the contrary we should, in a careful investigation, resort to statistics in order to secure greater definiteness and accuracy. The comparative number of the churches, the size of the police force, the number of saloons, would be noted and compared with the percentage of crime in order if possible to determine which of the above-mentioned circumstances is causally connected with the large number of criminals. That is, although we should not be likely to find fulfilled the strict requirements which this method makes, we should strengthen the inference by showing that definite quantitative relations exist, as indicated by

the statistics, between certain of the circumstances in question.

It is usual to speak of this method as that to which recourse must be had when it is impossible to employ experiment. As a matter of fact this illustration seems to show that the strict requirements of the method can never be realized except where experiment can be employed to isolate and control the conditions. In fields where this is impossible, it is necessary, as we have seen, to employ statistics as an instrument of analysis. Where the method is not supplemented by determining the relation of the various instances experimentally, or by making possible exact comparisons through the use of statistics, it can yield only vague and unsatisfactory results.

§ 66. **The Method of Concomitant Variations.** — In many other cases the discovery of certain forms of agreement or correspondence in the variations of phenomena, or groups of phenomena, enables us to detect a causal relation between them (cf. pp. 258 ff). The variations or changing states of all phenomena are events in time. Now when it is observed that certain of these events continue to show correspondences throughout a series of variations it is inferred that the conjunction is not accidental but indicates the existence of a causal connection. This correlation of events may be discovered through correspondences in temporal or spatial arrangement of phenomena, in their progression, or in changes of quality or quantity. The discovery of concomitant variations, however, is of importance in science, not merely because it assists us in determining what events

are related as causes and effects but also because the exact form of the causal relation can thereby be rendered more definite and satisfactory. For scientific knowledge the discovery of a 'general correspondence' between certain phenomena is not enough; it is necessary to obtain some *exact* expression of the relation between the two sets of variations. This is found by reducing the variations to terms of quantity through the application of a common unit of measurement. The law or ratio of the variations may then be expressed in numerical terms. Now the scientist tries to include in his statement of causa laws, whenever possible, precise information regarding the quantitative relations of the phenomena concerned. Indeed we may almost say that science does not exist until the quantitative aspects of phenomena are taken into account — until things are weighed and measured. The physicist does not think his work finished when he has proved that sound is produced by atmospheric vibrations. He carries on his analysis until he can discover the *quantitative* relations between the amplitude and velocity of the vibrations, and the loudness and pitch of the resulting tone.

Looking at two things with respect to the order and progression exhibited by their manner of appearance, then, we say that when their variations keep pace with each other they are in some way causally connected. What it is necessary to establish, in order to justify the inference to causal relationship, is that there is some definitely expressible relationship between the changes shown by the two series. Nothing is the cause of a phenomenon that varies when the

latter is constant, or is constant when it varies; or between whose changes and that of the phenomenon there is not some correspondence. It is not necessary however that the variations shown by the two series should always be in the same direction. One series may increase as the other increases, or the two series of changes may be in inverse ratio. The essential requirement is that there shall be some definite relationship clearly made out between the two series of events.

The following is Mill's statement of the canon: "*Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*"

As Jevons says,

> the illustrations of this law are infinitely numerous. Thus Mr. Joule, of Manchester, conclusively proved that friction is a cause of heat by expending exact quantities of force by rubbing one substance against another, and showed that the heat produced was exactly greater or less in proportion as the force was greater or less. We can apply the method to many cases which had previously been treated by the simple method of difference; thus instead of striking a bell in a complete vacuum, we can strike it with a very little air in the receiver of the air-pump, and we then hear a very faint sound which increases or decreases every time we increase or diminish the density of the air. This experiment conclusively satisfies any person that air is the cause of the transmission of sound.

Whenever, again, phenomena go through *Periodic Changes*,

> alternately increasing and decreasing, we should seek for other phenomena which go through changes in exactly the same periods, and these will probably be a connection of cause and effect. It is thus that the tides are proved to be due to the attraction of the moon and sun, because the periods of high and low, spring and neap tides, succeed each other in intervals corresponding to the *apparent* revolutions of those bodies round the earth.[1]

In employing this method it is of course hazardous to infer the existence of a universal law of correlation without examining in some detail the nature of the concomitant variations. In general the more definitely the relationship can be shown in a considerable number of cases, the more ground there is for the conclusion that the conjunction is not accidental. It is also necessary that observations should be extended over a considerable range in order to determine whether the supposed law of correlation has any limits, and if so how they are to be defined. For example, in Weber's law we have an exact expression for the correlation of the quantity of the stimulus in the case of the various sense organs and the intensity of the resulting sensation. But in every case this exact correlation of stimulus and sensation has an upper and lower limit, beyond which it either changes its character or ceases altogether.

The close and almost inseparable connection of the different methods in actual use is clearly evident. In many fields it is only through experiment that the fact of correspondences between phenomena can be brought to light,

[1] Jevons, *Lessons in Logic*, pp. 249–251.

and the character and law of their correlations exactly determined. But to introduce experiment for these purposes is of course to supplement the method of Concomitant Variations by the method of Difference. Similarly in performing experiments where it is impossible to withdraw a certain element, and thus by comparison to note what its cause or effect is, as the strict canon of Difference requires, we may be able to isolate the element practically by causing it to vary while other circumstances are kept constant. It is then possible to note the variations in the corresponding series and thus to determine what is causally correlated with the element in question. If for example the problem were to determine the effect of moisture on growing plants it would of course be impossible to eliminate moisture entirely without killing the plant and putting an end to the experiment. But by varying the amount of moisture and noting concomitant changes in the plant, both methods of analysis are combined.

§ 67. **The Method of Residues.** — In general, this method calls attention to any remainder or residue which is left over after other portions of a complex phenomenon have been explained. There are two results of its use which may be discussed separately.

(*a*) Its application to a complex phenomenon which is the result of several causes often enables us to determine what part each of these causes plays in the determination of the whole fact under consideration. Mill's fifth canon seems to apply to this case. It is as follows: "*Subduct from any phenomenon such part as is known by previous inductions to*

be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents." Thus if it is known that the complex phenomenon BAC is the result of *bac*, and if it is further known that *a* is the cause of A, and *b* of B, it follows by subtraction that the residue still unexplained, C, is caused by *c*, the remaining antecedent.

Of course the application of this method in concrete cases does not usually resolve itself into such a simple process of subtraction. It requires work — 'previous inductions', as Mill says — to determine what are the whole number of antecedents in any case, as well as to isolate the various antecedents so as to determine exactly what part of the effect is to be ascribed to each one. This may be illustrated by an example: after my student's lamp has been lighted two hours I find the thermometer has risen from 65° to 70° Fahr. The phenomenon to be explained then is the additional 5° of heat. There is no fire, and it seems that the increase in temperature must be due to the lamp, and the heat given off from my body during this period. Suppose that the lamp is burned for the same length of time while the room is unoccupied, all other conditions remaining the same, and that the thermometer shows an increase of 4° in the temperature. By subtraction we could conclude that the heat given off by the body on the former occasion was the cause of the additional degree of temperature.

To carry the process of analysis a step further. Let us suppose that a half pint of oil, which is composed of hydro-

gen and carbon, has been consumed. We could determine, by measuring the heat produced by the oxidation of the exact amount of carbon contained in one-half a pint of oil, what quantity of heat is due to the combustion of the carbon contained in the oil, and, by subtraction, what must be ascribed to the burning of the hydrogen.[1]

(*b*) The second case in which this method may be applied is where there is an unexplained remainder or residue left over after the result of all the known causes has been calculated. Mill does not distinguish between such instances and the method of simple subtraction discussed above. Since however the cause must explain the whole of the effect the method of residues enjoins us to continue the search for explanation. *When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.* If for example it were found by actual measurement that the heat produced by the lamp, and by the body of the occupant, were not sufficient to account for the change in temperature of the room, it would be necessary to seek for some further cause to account for this unexpected remainder.

This method can scarcely be said to be more than a demand for complete and precise explanation. The attempt, however, to account for unexplained residues has led to many extremely important discoveries in science. Residual phenomena are often so obscure and appear so uninterest-

[1] This is not strictly correct, for it leaves out of account the heat generated by the *chemical combination* of the carbon and hydrogen. It may therefore serve to illustrate a case where the method of Residues breaks down.

ing and unimportant to the ordinary mind that they are passed over without explanation. It usually requires the eye of a scientific genius to see the importance of things which appear trivial and unessential.

A very striking example of the application of this method is afforded by the history of the discovery of the planet Neptune. In 1781 Sir William Herschel discovered Uranus, a new planet, moving outside all the others. When its orbit came to be calculated it was found that it did not move as it might be expected to do according to the theory of gravitation. That is, the attraction of the sun and the known planets did not account for the path it took: it moved outwards into space farther than it ought to have done. It was evident that either some mistake must have been made in the observation of the astronomers, or some unknown body must be dragging it out of its course. No traces of any such planet could be perceived, and the problem remained unsolved. In 1843 an astronomer named Adams undertook to work out the movements of Uranus, to discover if possible the position of the body which was pulling it out of what would otherwise be its proper path, the attractions exercised by the sun and the planets in their different positions, and to show what effect they would have in determining the orbit of Uranus. Whenever the planet was deflected outwards it was necessary to show where the body was situated which was thus influencing it. In 1845 he was able to send a paper to the astronomer royal at Greenwich informing him in what quarter of the heavens the new planet should be observed. When the

discovery was afterwards made it was proved that his calculations were almost exactly correct. A failure on the part of the astronomer royal to coöperate by looking through his telescope for the planet gave the prior right of discovery to a Frenchman named Leverrier. The latter worked out his calculations in the same way as Adams, and obtained almost exactly the same results. He sent these results to Professor Galle of the Berlin University on the 23d September, 1846, asking him to look in the part of the heavens which he indicated. That same evening, by following out the directions, the planet was discovered in almost the exact spot predicted.

§ 68. **Final Estimate of the Methods.** — In the course of this chapter we have had occasion to point out the limitations and defects of each of the five methods severally. We have now to estimate their collective value as an aid to inductive inference in general. Mill himself claimed that they "provide rules and models (such as the Syllogism and its rules are for ratiocination) to which if inductive arguments conform those arguments are conclusive, and not otherwise." [1] But as Latta and Macbeath point out, "such claims on their behalf are exaggerated and tend to bring them into disrepute. . . . The data of observation have to be analyzed, and causes have to be suggested before they can be applied; just as premises have to be taken for granted and arguments put into strict logical form before the rules of the syllogism can be applied. When Mill tries to apply them to concrete phenomena the results are not

[1] *Logic*, Bk. III, Ch. II, § 5

very happy. Moreover, even when this preliminary work has been completed, the methods do not *prove* causal connections. They can only give further confirmation or added probability to the connections already suggested.[1] . . . their main value is to suggest causes which can be further tested by other methods." [2]

Mill's comparison of his rules of causal connection with the rules of the syllogism only serves to give his own case away, once the strictly limited significance of the latter has been recognized.[3] In both cases the claim to have reduced reasoning to a quasi-mechanical procedure, according to criteria established once and for all, has failed of realization. It is not to any simple set of inflexible rules, but only to broad principles, capable of reinterpretation and development *pari passu* with the growth of knowledge itself, that we may look for guidance in our reasonings, be they inductive or deductive. On this point the analogy of logic with ethics is instructive. Some moralists have held that it is the business of ethics to provide us with a set of rules for the guidance of conduct at every step and under all possible circumstances. Any such scheme is bound to break down when confronted with certain novel situations which will inevitably arise in the course of time. And leading jurists, such as Dean Pound and Justice Cardozo, are always pointing out that the same thing is true in respect to laws. Judges must look upon laws as flexible principles

[1] Cf. above, pp. 283, 284, 285, 290, 295, 299, 301, 302.

[2] Latta and Macbeath, *The Elements of Logic*, pp. 325, 326. For further criticism see Sigwart, *Logic*, and Bradley, *Principles of Logic*.

[3] See above, Ch. XI.

rather than as inflexible rules. But these failures will be disheartening only to the proponents of such ultra-conservatism, either in the sphere of scientific inference or in that of human conduct. To every one else the ability to transcend rules and schedules will appear as a revelation, not of mental or moral impotence, but rather of an increase in intellectual power and freedom.

And quite irrespective of the value of Mill's methods for the discovery of causal relations, they certainly cannot be applied to the discovery of other types of relations. Here comparison with the syllogism is again instructive. Just as deduction is a broader term than syllogism, so induction is by no means limited to the sphere of causal connections. As a good empiricist Mill himself was interested in maintaining that induction is as typical of the mathematical sciences as it is of physics and chemistry. And many other logicians also support this thesis.[1] But the terms cause and effect do not appear at all in the vocabulary of the mathematician, while to the biologist and other scientists they are subordinate to other categories, such as that of purposiveness.

In the following chapters we shall see how the shortcomings inherent in Mill's methods are made good by other modes of procedure.

EXERCISES (XVI)

1. Is the causal relation identical with a relation of invariable temporal sequence? Defend your answer.

2. What is the problem which Mill's methods were intended to solve?

[1] See H. R. Smart, *The Logic of Science*, especially Ch. IV.

3. What is meant by the phrases, Plurality of Causes, and Reciprocity of Causes?

4. How does the Method of Agreement differ from induction by simple enumeration?

5. How might the canons of Agreement and Difference respectively be stated negatively, as principles of elimination? Would this statement do full justice to the inductive procedure involved?

6. What is meant by calling the Method of Difference a method of scientific experiment?

7. How is one weakness of the Method of Agreement overcome by the Joint Method?

8. Explain carefully, using an original example, how you would apply the Joint Method.

9. What is the relation of the Method of Concomitant Variations to the Method of Difference?

10. How is the former Method affected by the operation of permanent causes, such as gravitation?

11. Mention some discoveries to which the investigation of unexplained residues has led.

12. Consider the difference in meaning and significance of the term cause as employed in ordinary experience and in the sciences, and indicate what bearing this has on the utility of the Methods.

13. Why is the use of alphabetic symbols to represent causal relations misleading? Explain fully.

NOTE: — *Examples illustrative of these Methods will be found amongst the exercises at the end of the volume.*

CHAPTER XVII

ANALOGY

§ 69. **Explanation by Analogy.** — An **Analogy** may be defined in general terms as an agreement, resemblance, or proportion between the relations of things to one another, or between the things themselves. Thus it might be said that there is an analogy between the relations of a ruler to his people and those of the captain of a vessel to members of his crew. Or an analogy might be said to exist simply between a ruler and a captain, or between a state and a ship. In logic, analogy is used more specifically as a form of reasoning in which, from the resemblances of two or more things in certain respects, their likeness in other respects is inferred.

The tendency to note resemblances and to assume that things alike in certain respects are alike in all, is present from the first in all stages of thinking. We have seen (§ 54) that this principle guides inductive inquiry by furnishing suggestions as to what may be expected when new facts and conditions are met with. We seek to assimilate what is new to that with which we are already familiar. But in noting, in our earlier discussion, the operation of this principle, no detailed description of its principles was given, or any adequate account of the part it plays in organizing experience. In this chapter emphasis is laid more particularly on the function that Analogy performs at a somewhat advanced stage

of inductive inquiry, in leading on to the higher generalizations of science. At a lower level the connections and relations suggested by Analogy are of a factual and descriptive character. For example, Analogy might suggest in a particular case that the severe frost is the cause of the bursting of water pipes, without affording any clear understanding of the universal law through which these things are connected. In more advanced stages of knowledge, however, Analogy is used consciously and critically as a means of deriving general laws and principles of explanation. In proceeding to the discussion of this more explicit use of Analogy we may be said to be passing from Description to Explanation. But as has already been pointed out, no hard and fast line can be drawn between the determination of the nature and connection of facts, and their explanation. The task which our thought is called upon to perform is to transform obscurely known and isolated facts into an orderly and consistent system of knowledge, and this process is continuous throughout. But keeping this in mind, one may still say it is necessary, in the first place, for the facts to be thoroughly analyzed and carefully examined; and secondly, for them to be grouped together according to some general principle or principles which shall make clear and intelligible the relations in which they stand to one another.

To explain is just to show that some fact or group of facts is related in an orderly way to some other fact or group with which we are acquainted. So far as the methods we have discussed enable us to establish connections between events, they may fairly claim to be methods of explanation. Never-

theless, although the difference between these methods, and those of explanation in terms of wider generalizations, is one of degree rather than of essential nature, it is important to keep it in mind.

The principle of Analogy is resemblance. The phenomenon to be explained is connected with some more familiar occurrence through a perceived or imagined likeness between the two cases. All our first rude classifications and explanations are based on this principle. In the early stages of the history of the race everything was explained on the analogy of human actions. All natural events, that is, were supposed to be produced by superhuman agents, who were however endowed with essentially the same qualities as man. In the thunder, the men of a primitive age heard the voice of a god. An eclipse of the sun or moon was interpreted as a divine sign or warning. When the sea became tempestuous and lashed its shores they believed that the sea-god was angry. In every case they interpreted these mysterious happenings of nature by referring them to causes similar in character to those which they best understood as effective forces — the motives and volitions of themselves and their fellows.

The principle of analogy is employed in the same way in modern times. It is true that we no longer think that natural events are directly caused by the action of some spiritual agent more or less like ourselves. But when we endeavor to show that the phenomena we are trying to explain are similar in important respects to some group of facts with whose mode of operation we are familiar,

we proceed by analogy. On the basis of this similarity we argue that the phenomena with which we are dealing probably have the same properties, or operate in the same way, or are governed by the same laws, as the better-known facts that they resemble. The formula of analogy may be stated in this way: Two things resemble each other in one or more respects, they are therefore of the same general type or character; it follows that a certain proposition true of the one is probably true of the other. The following example of analogy has been frequently used as an illustration:—

> We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus, and Mercury. They all revolve round the sun, as the earth does, although at different distances and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve around their axes like the earth, and by that means must have a like succession of day and night. Some of them have moons that serve to give them light in the absence of the sun, as our moon does to us. They are all in their motions subject to the same law of gravitation as the earth is. From all this similitude, it is not unreasonable to think that those planets may, like our earth, be the habitation of various orders of living creatures.[1]

The word 'analogy' at the present time is somewhat loosely used for any mark of similarity or resemblance which enables us to reason from one thing to another. As already noted, the term is also applied either to a likeness between two things, or a likeness between certain

[1] Reid, *Intellectual Powers of Man*, Essay I, Ch. III.

relations of things. In the latter case there is of course a proportion expressed, as when it is said that the relation of a clergyman to his parishioners is analogous to that of a physician to his patients. The purpose of such comparisons is to afford a basis for inferring that the rights or duties that exist in the one case obtain also in the other. In such cases, however, we have always to ask if there are not *differences*, as well as likenesses, in the two sets of relations. This employment of analogy is more strictly that which was noted and defined by Aristotle.

> The original word ἀναλογία, as employed by Aristotle, corresponds to the word Proportion in Arithmetic; it signifies an equality of ratios, ἰσότης λόγων: two compared with four is analogous to four compared with eight. There is something of the same meaning in the technical use of the word in physiology, where it is used to signify similarity of function as distinguished from similarity of structure, which is called homology; thus the tail of a whale is analogous to the tail of a fish, inasmuch as it is similarly used for motion, but is homologous with the hind legs of a quadruped. A man's arms are homologous with a horse's fore legs, but they are not analogous, inasmuch as they are not used for progression.[1]

Apart from these technical uses what is known as analogical reasoning may perhaps be best defined as an argument from similar instances. In analogy we do not stop to work out a law of connection between phenomena by comparing a number of cases, or by using any of the ordinary inductive canons. But finding a striking resemblance between

[1] Minto, *Logic, Inductive and Deductive*, p. 367.

some circumstance — relation, quality, arrangement, function, etc. — in the phenomena to be explained, and some phenomena with which we are already acquainted, we use the latter as a basis for conclusions about the former. Analogy is thus an argument from examples or instances, its value depending upon the real identity in some important aspect of the cases compared. When however our thought is able to extend to a new case, or set of cases, some general law or principle with whose operation it is already acquainted in other instances, we have passed beyond analogy to a higher form of explanation. In the former case we argue from the resemblance of instances; in the latter the thread binding the new instance with the old is the identity of a general principle.

§ 70. **Analogy as Suggestive of Explanatory Hypotheses.** — We have shown above that analogical reasoning depends on the resemblance existing between individual cases or instances, and that it does not itself succeed in formulating any general law or principle. The next section will show in more detail in what respects the principle of analogy falls short, and why, taken by itself, it can only be regarded as incomplete explanation. Here we have to notice the important part which it plays in suggesting laws and principles. Although analogy 'sticks in the particular instances', it leads the mind on to general laws and explanatory theories. It is thus of the greatest importance as a necessary stage on the way to complete explanation.

When we are able to discover some general resemblance

between a group of phenomena which we are interested to explain, and another group whose principle of operation we already understand, our thought strives to extend the known principle and to bring the new facts under it. The unknown or unexplained facts are thus brought under a known law. It is of course true that the application of the law to a new set of facts broadens our conception of its scope, and often requires us to state it in a more adequate way. Thus the analogy Newton is said to have perceived between the heavenly bodies falling through space and the falling of the apple towards the ground led to the formulation in exact mathematical terms of the universal law of gravitation. Our knowledge of the various functions of plants — digestion, reproduction, etc. — has been obtained by ascribing to the various organs of the plant purposes analogous to those which are fulfilled by the parts of animal bodies. And in turn the study of plant physiology has thrown light upon animal physiology and enlarged and modified many of its theories. Again, the explanation of many geological changes — the wearing away of rocks, the formation of deltas or of great ravines, of vegetable mould, etc. — is facilitated by a discovery of their analogy with familiar events which happen constantly before our eyes. In mathematical reasoning, too, analogy has a very considerable part to play. As Poincaré puts it: —

> . . . the mathematical facts worthy of being studied are those which, by their analogy with other facts, are capable of leading us to the knowledge of a mathematical law, just as experimental facts lead us to the knowledge of a physical law.

They are those which reveal to us unsuspected kinships between facts, long known, but wrongly believed to be strangers to one another.[1]

An extremely interesting instance of the part which analogy plays in suggesting possible explanations is found in the account of the discovery of the principle of Natural Selection given by Darwin in his *Autobiography*. In 1837 Darwin opened a note-book for the purpose of recording all facts in any way connected with the variation of species in nature and under domestication. He first investigated the variations of plants and animals which are produced under domestication, by printed inquiries, by conversation with skilful breeders, and by extensive reading. "I soon found", he says, "that selection was the keystone of man's success in making useful races of plants and animals." When useful or pleasing varieties of plants or animals occur, the gardener or breeder preserves them, and their peculiar qualities are transmitted to their offspring. And in a number of generations these qualities become more pronounced through accumulation. The differences between varieties of the same species of domesticated animals — varieties which are as different, for example, as the mastiff and Skye terrier — are due to the selective agency of man. But is there anything analogous that takes place on an indefinitely larger scale in nature? If so, what is it which plays the part of the gardener or breeder, and preserves certain varieties?

When Darwin had reached this point in his investigations and had come to appreciate what selection could do, he

[1] *Foundations of Science* p. 386.

happened to read Malthus's book, *On Population.* The purpose of this book was to dispel the optimistic ideas of some of the writers of the eighteenth century who looked for the speedy realization of social well-being and happiness. Such an ideal is impossible of fulfilment, said Malthus, because of the inevitable tendency of population to increase faster than the supply of food. Human beings increase in a geometrical ratio; the means of subsistence, at best, only by an arithmetical ratio. The population will thus constantly tend to exceed the limit of the food supply, and will be kept in check only by starvation. A constant struggle for food is the lot, then, to which each individual is doomed in virtue of this law. Darwin's observations of the rate at which plants and animals tend to reproduce their kind led him at once to extend Malthus's principle to the whole of nature. The fecundity of natural beings leads to a struggle for existence, not merely among men, but throughout the whole organic world. And if there is a struggle, we have natural selection or the survival of the fittest. Darwin saw "that natural selection was the inevitable result of the rapid increase of all organic beings." It is not difficult to see that this discovery was the result of Darwin's wonderful power of perceiving analogies between different classes of facts. His genius led him to recognize first the resemblance of the variations of species in nature to the more familiar variations that go on among domesticated plants and animals. And secondly he perceived that the competition for the means of subsistence, which the pressure of population imposes upon the members of the human race, is simply one

phase of 'the struggle for existence' going on everywhere throughout the organic world.

§ 71. **The Incompleteness of Analogical Reasoning.** — The most striking feature of analogical arguments is found in the fact that they yield only probable conclusions. The reason for this is not far to seek. For as has been already shown, analogy is a method of reasoning from one particular case to another on the basis of some imagined or perceived similarity between the two cases. But complete logical demonstration, or certainty, is attained only when the new fact or group of facts is really and essentially united by means of some general principle with what is already known. There is no genuine inference from 'particular to particular', as Mill supposed. Inference, as has been well said, always 'proceeds through a universal'. It is the universal implied in the common name, or vaguely present in the mind of the reasoner, which really carries the inference in cases where conclusions appear to be drawn from a particular case. When one reasons that food or drink which has made A ill will produce the same result in B, it is the universal nature of human beings on which the inference is based. In the case of Analogy the inference lacks certainty because the universal nature is not completely analyzed or defined. Instead, it is more or less vaguely assumed in the form of external likeness or resemblance.

But although Analogy yields only probable conclusions, it must not be forgotten that 'probability' is not a fixed quantity. An argument from analogy may have any degree of value from zero almost up to the limit of complete logical

certainty. To *fully* explain or demonstrate any fact we are obliged, I think, to go beyond analogy, and to verify its conclusions by bringing them into relation to a general principle. It is evident that the value of an analogical argument will depend upon the nature of the resemblance taken as the basis of inference. In general it is true that the greater the resemblance between the two cases, the more certainly can we reason from one to the other. This is not to say, however, that the value of the conclusion is in direct proportion to the number of points of resemblance that can be discovered. For example, we might reason: These two men are of the same height, of the same age, live in the same house, come from the same town; the one man stands well in his classes, therefore the other probably does so also. If the *number* of points of resemblance were the essential thing, the argument ought to possess some weight, but it is clear that it has none. The difficulty is that none of the resemblances mentioned is fundamental, or in any way essential to the real nature of the things compared. If we knew that the two men were similar in character, this one characteristic would be worth more, as a basis for the conclusion, than all the circumstances which we have mentioned combined.

It is true, then as Bosanquet remarked, that in analogical reasoning we must *weigh* the points of resemblance rather than count them.[1] Other things being equal, the more points of resemblance we can make out the better; but if these are to contribute at all to the certainty of the conclusion they

[1] *Logic*, Vol. II, 2nd ed., p. 99.

must represent some deep-lying characteristic of the things compared. In general it must be said that it is only experience that can inform us what resemblances are fundamental, and what merely external. Systematic knowledge in any field enables us to separate the essential from the accidental. And what is perhaps a corollary from this, it must not be forgotten that the value of an inference from analogy depends largely upon the amount of intellectual insight possessed by the mind which makes it. The ordinary mind, at least in its undisciplined and untutored condition, regards all things as of equal importance. It is therefore led away by the strongest stimulus — by striking external and accidental resemblances — as is well shown by the readiness with which such minds are carried away by the fallacies of figurative or analogical language. On the other hand a scientific genius whose mind is well stored with facts and who is gifted in addition with imagination, is able to penetrate beneath the surface and to apprehend the real or fundamental resemblance. His imagination enables him to see beyond the chaos of the particular facts and to detect the underlying principle by means of which these facts can be connected and systematized.

Analogy thus becomes deepened until it passes from the stage of a mere argument from particular to particular, to the perception of a general law which includes the individual instance. But no such direct insight can claim the title of knowledge until it is tried and tested by the facts. The guesses of scientific men unfortunately often prove mistaken. It is always necessary that fancy shall be confronted

with facts. Even Darwin's magnificent analogical inference was nothing more than an hypothesis, as he himself well understood, until its power of explaining the facts of organic life was demonstrated. We have now to explain in the next chapter the methods by which such guesses are tested.

EXERCISES (XVII)

1. Why do we include Analogy among the methods of explanation?

2. State and illustrate three senses in which the word analogy is used.

3. What does Mill mean by calling Analogy 'a mere sign-post'?

4. Evaluate the analogies given below, comparing them as to number of points of resemblance, and as to the weight of these points. How far do they go in establishing the conclusions?

(*a*) There are seven windows in the head, two nostrils, two eyes, two ears, and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven (from a writer of the seventeenth century).

(*b*) We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus and Mercury. They all revolve round the sun, as the earth does, although at different distances, and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve round their axis like the earth, and, by that means, must have a like succession of day and night. Some of them have moons, that serve to give them light in the absence of the sun, as our moon does to us. They are all in their motions, subject to the same law of gravitation, as the earth is. From all this similitude, it is not unreasonable to think, that these planets may, like our earth, be the habitation of various orders of living creatures (Thomas Reid, quoted by Stebbing, *A Modern Introduction to Logic*, p. 252).

(*c*) An early problem of geometry was the calculation of the area of a quadrilateral plane figure in terms of the lengths of its sides. In the case of a square of length of side a it was easy to discover that its area was $a.a$, or a^2. Analogously, it was easy to find that in the case of *any* rectangle of sides a and b, the area was given by the product $a.b$. By an extension of this same analogy it was sought to prove that the area of an isosceles triangle would be $a.b/2$, but here the analogy broke down (adapted from Smart, *The Logic of Science*, p. 107).

5. In what figure of the syllogism does an argument from Analogy naturally fall? Is the argument formally valid, and if not, to what syllogistic fallacy does it correspond?

6. The history of science is full of instances of analogical reasonings which have led to the formulation of hypotheses and explanatory theories. Mention some of these.

CHAPTER XVIII

THE USE OF HYPOTHESES

§ 72. **Reasoning from an Hypothesis.** — An hypothesis, taken in its most general sense, is a guess or supposition as to the existence of some fact or law which will serve to explain a fact or connection of facts already known to exist. It is thus an expression of the tendency of the mind to leave nothing standing in isolation, but to 'explain' the various parts of experience by bringing them into relation with one another. 'Theory' is another word that is often used as equivalent to hypothesis. Strictly speaking, however, it is better usage to employ the term 'hypothesis' for the unverified, or only partially verified guess, and to reserve 'theory' for the hypothesis that has been more completely demonstrated. This distinction, however, is not usually maintained, and even in scientific writings the terms 'theory' and 'hypothesis' are used interchangeably. Nevertheless it is necessary to distinguish in some way the 'mere hypothesis', or supposition, which is often as likely to be false as true, from the hypothesis which has been established by proof.

It is important to remember that it is not only in solving scientific problems that we employ hypotheses. In our ordinary experience we are constantly trying to imagine the most likely explanation of facts which we perceive through

the senses. If for example one should find on returning to one's room that a pane of glass had been broken, one would straightway set about finding some explanation of this occurrence. One might perhaps first imagine that a stone or something of the kind had been thrown against it. Acting on this supposition, one would look for the stone in the room. If it were found there, the hypothesis would be confirmed; if no traces of it could be discovered, and if, moreover, on examination the glass proved to be shattered in a way that would probably not result from the projection of a stone against it, our first hypothesis would have to be abandoned. We should then make another guess — perhaps that the outside blind had been violently closed by the wind — and again examine the facts to see if they gave any support to this supposition. We are constantly making hypotheses of this character to explain the phenomena we meet with in everyday experience. If we find a stream swollen, we conclude that it must have rained in some part of the country drained by the stream. If a man has typhoid fever, we are pretty sure to guess that he has been drinking impure water. We no sooner perceive something unusual or striking than we begin to guess out, as it were, its explanation. The formation of hypotheses, then, is simply the mind's response to the demand for explanation.

The examples given above illustrate what may be called the popular, as opposed to the scientific use of hypotheses. In these cases the hypothesis assumes the existence of a particular thing or event as that through which the phenomenon in question is to be explained. The 'law' at which the

induction arrives is that of a causal connection of phenomena taken in a descriptive or factual way. Analysis is not carried on to reach a genuinely explanatory hypothesis, as it would be in a strictly scientific investigation. Such an explanatory hypothesis would not point to any particular phenomenon as a 'cause', but would state as a law certain permanent forms of relation in which things and events stand, and under which the phenomenon in question is assumed to fall. Think of the difference in character between the hypothesis that the window was broken by the slamming of the blind, and, for example, Newton's law of Gravitation, or the vast generalization of facts included in Darwin's law of Natural Selection.

Nevertheless it cannot be maintained that the distinction is in any sense absolute between the hypothesis of a fact and the hypothesis of a general law of relation. What is an hypothesis at one stage becomes, when verified, for further investigation a fact or starting point. Between the popular and the scientific use of hypotheses there are important differences of degree, as has been pointed out. In discussing the use of hypotheses in this chapter we shall have in mind primarily the reflective and critical procedure through which certain conceptions are defined and tested as instruments for the colligation of facts. We shall thus be studying, in its highest and most explicit form, the function that guides Induction from its earliest beginnings.

It is worth noticing that it is only unusual or striking events, or those in which they have some practical concern, which attract the attention of the majority of mankind, and

lead them to form explanatory hypotheses. What is familiar, or of no practical importance, does not usually awaken curiosity. Indeed, in a great many cases, such phenomena are not observed at all. But the great scientist is distinguished, one may say, by his intellectual curiosity. He tries to understand phenomena which the ordinary mind neglects and simply takes for granted. He has questions in his mind with regard to familiar things which he wishes to have answered, guesses he is desirous of having proved or disproved. Unless the mind has some question to answer, or theory to test, it is impossible to see any significance in an experiment. In other words every experiment must have a purpose, and the purpose is to get some information that will help us to answer a question which we bring with us to the investigation.

In the actual process of acquiring knowledge, then, observation and theorizing go hand in hand. Unless we go to nature with something in our mind, we are not likely to learn much. As a rule we see only what we look for. Francis Darwin says of his father: —

> He often said that no one could be a good observer unless he were an active theorizer. This brings me back to what I said about his instinct for arresting exceptions: It were as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact become magnified into importance. In this way it naturally happened that many untenable theories occurred to him, but fortunately his richness of imagination was

> equalled by his power of judging and condemning the thoughts which occurred to him. He was just to his theories and did not condemn them unheard; and so it happened that he was willing to test what would seem to most people not at all worth testing. These rather wild trials he called 'fool's experiments', and enjoyed exceedingly. As an example, I may mention, that finding the cotyledons of Biophytum to be highly sensitive to vibrations of the table, he fancied that they might perceive the vibrations of sound, and therefore made me play my bassoon close to a plant.[1]

A good example of how essential theories are for an observer, and how blind he may be to what he is not looking for, is found in the work from which we have just quoted. In the brief autobiography contained in the first volume, Darwin tells of a geological trip through Wales which he took while a student at Cambridge, in company with Sedgwick, the professor of geology. It must be remembered that this was before Agassiz had come forward with his theory of a glacial period in the world's history. Darwin writes: —

> We spent many hours in Cwm Idwal, examining all the rocks with supreme care, as Sedgwick was anxious to find fossils in them; but neither of us saw a trace of the wonderful glacial phenomena all around us; we did not notice the plainly scored rocks, the perched boulders, the lateral and terminal moraines. Yet these phenomena are so conspicuous that as I declared in a paper published many years afterward in the *Philosophical Magazine*, a house burnt down by fire did not

[1] *Life and Letters of Charles Darwin*, Vol. I, p. 126.

tell its story more plainly than did this valley. If it had been filled by a glacier, the phenomena would have been less distinct than they are now.[1]

§ 73. **Formation of Hypotheses.** — We are now ready to consider a little more closely the formation of hypotheses or theories. In the first place it is to be noticed that hypotheses are not received from without through sense-perception, but are made by the mind. They are the creations of the imagination. A good theorizer, like a poet, is in a certain sense born, not made. The man to whom 'nothing ever occurs', whose intellectual processes are never lit up with a spark of imagination, is unlikely to make any important discoveries. It has been by a flash of scientific genius, by imaginative insight which we may almost call inspiration, that great scientific theories have been discovered. Not even a scientific genius, however, can afford to neglect the facts. But guided by accurate observation, the scientific imagination tries to invent some law or principle which will serve to connect and explain facts. Tyndall has an essay on "The Scientific Use of the Imagination" from which we may quote a short passage.

> With accurate experiment and observation to work upon, imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the prepared imagination. . . . Out of the facts of chemistry the constructive imagination of Dalton formed the atomic theory. Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, pre-

[1] *Life and Letters of Charles Darwin*, Vol. I, p. 49.

ceding, accompanying, and guiding all his experiments. His strength and fertility as a discoverer are to be referred in great part to the stimulus of the imagination. Scientific men fight shy of the word because of its ultra-scientific connotations; but the fact is, that without the exercise of this power, our knowledge of nature would be a mere tabulation of co-existences and sequences.[1]

In speaking of hypotheses as 'guesses', or 'creations of the imagination', their dependence upon facts must not be forgotten. It is only when the phenomena to be explained have been carefully observed that our guesses at their explanation are likely to be of value. It is well known that a considerable amount of knowledge is usually required to ask an intelligent question. And in the same way the mind must be well stored with facts, in order to render our hypothetical explanations worthy of consideration. Indeed, observation of facts and the formation of theories go hand in hand, and naturally assist each other. We have already spoken of the lack of theory which makes us blind to facts that seem to lie directly before us. But we have perhaps not yet emphasized sufficiently the dependence of theories upon the facts of observation. The process of explanation may be described as a fitting together of the facts given by observation, with the explanatory theories that the mind originates. The theory with which we start enables us to ask questions, and leads us to scrutinize the phenomena to be explained; while the latter react upon the theory, and cause it to undergo constant modification. Neither the

[1] *Fragments of Science*, p. 104.

'theory' nor the facts are to be regarded as fixed and unchanging; both are constantly changing in relation to each other as the investigation proceeds. The account of Darwin's discovery of the principle of 'the survival of the fittest' is a good illustration of an hypothesis constructed by a constant dependence upon the facts during every step of its progress.

§ 74. **The Proof of an Hypothesis.** — We have discussed the way in which hypotheses are formed, but as yet have said nothing regarding the means of determining their truth or falsity. But to form hypotheses is usually easy, to verify them is often exceedingly difficult. The scientific worker constantly finds that theories he has formed, on the basis of analogies or otherwise, cannot be verified, and must therefore be discarded. It is not only essential that a scientific investigator shall possess a mind fertile in ideas; he must also love truth more than any theory, no matter how interesting or attractive it may appear. In behalf of truth, every theory must be subjected to the most thorough and searching tests possible; if it is not borne out by facts, it must be at once discarded. What now is the general method of procedure in testing an hypothesis? How do we proceed to compare our theories with the facts? Two steps or stages may be distinguished in this process: (1) We assume that the hypothesis is true, and proceed to show what are the necessary results following from it. In doing this we proceed deductively; that is, assuming the truth of the hypothesis, we reason out what consequences must follow from it in accordance with laws whose mode of action we already

know. (2) The conclusions thus reached are compared with the actual facts, as given to us directly in perception, or as determined by experiment. If they are found to agree with these, the hypothesis is regarded as true; if they do not agree, it becomes necessary to discard the hypothesis, or to modify it in some way suggested by the results so far obtained by the investigation.

This procedure may become clearer by considering some concrete examples. We may first take an illustration of what has been called the popular use of an hypothesis. If we were to come on the campus some morning and find that several branches had been broken from one of the trees, we should naturally try to explain this circumstance by making some hypothesis. Perhaps the first thing which would occur to us would be that there had been a violent wind storm. The hypothesis having been made, the next step would be to look around to see if it could be verified. 'If there has been a cyclone', we might argue, 'there should be other signs of its presence; we should find broken twigs and blown leaves lying about, and all the trees should present a storm-tossed appearance'. If observation showed that these things were actually present, we would consider our hypothesis so far confirmed. But if not, our first guess would be disproved, and it would be necessary to look about for another explanation. In this case, the second hypothesis, being based on a better analysis of the facts, would be more likely to prove correct than the first. But the process might have to be continued through several steps.

An excellent illustration of the way in which a scientific

hypothesis may be rendered more certain and at the same time more comprehensive and definite is found in the history of the experiments by which it was proved that the atmosphere has weight. Galileo noticed that water will rise in a pump only about thirty-three feet. He could not find out, however, why it was that the water stopped at this point. After his death his friend and pupil Torricelli took up the problem, and asked himself: Why does the water rise at all? It then occurred to him that air must weigh something, and that it might be this weight on the surface of the water which forced the water up the pump when there was no air pressing it down. Now if this were so, he reasoned, the weight of the air ought to lift mercury, which is fourteen times heavier than water, to one-fourteenth of the height. So he took some mercury, and filling a tube about thirty-four inches long, turned it upside down into a basin of mercury which was open and therefore under the pressure of the atmosphere. The mercury began to settle in the tube, and finally rested at a height of thirty inches. Torricelli had thus invented the barometer, an instrument that would measure the weight of the atmosphere. It was afterwards suggested by the famous French writer, Pascal, that at the top of a high mountain, where there is less air pressing downwards, the column of mercury should fall considerably if the atmosphere were really what caused the water and the mercury to rise. When this experiment was made by carrying the barometer to the top of a mountain called the Puy de Dôme, the mercury fell nearly three inches. Still further confirmation of Torricelli's theory was afforded

by the discoveries of Otto Guericke of Magdeburg. In 1650 Guericke invented the air-pump. The first use which he made of his new invention was to show that the atmosphere is pressing down upon us heavily and equally in all directions. He fitted closely together two metal hemispheres and exhausted the air between them by means of his pump. It was found that the pressure of the atmosphere was so great that it took a great force to separate the hemispheres.

To establish a scientific theory, then, there is necessary not only a ready imagination, but also patience and perseverance in the careful deduction of the consequences of the theory, and the comparison of the results thus obtained with the actual facts. Scientific work also demands the utmost candor and openness of mind on the part of those who engage in it. One must be willing to abandon any theory as soon as it is found to disagree with the facts. And this is by no means an easy thing to do. When one has a theory which suffices for nearly all the facts, there is always a temptation to cling to it, and to neglect or explain away any troublesome or contradictory facts. There is no doubt that the scientific explanations which have become accepted and established were not the ideas which first happened to occur to the men with whose names they are associated. When Newton first attempted to work out the verification of the gravitation hypothesis he used the most accurate measurements he could obtain regarding the size of the earth. But in calculating on this basis the pull of the earth on the moon, and the consequent deflection of

the moon from the straight line, his results came out wrong. That is, the moon moved more slowly than it ought to move according to his theory. The difference was not great, but Newton could not overlook this lack of agreement with the observed facts. He put the whole matter aside; and it was only when he heard, sixteen years later, that Picart had discovered from new and more accurate measurements that the earth was larger than had been supposed, that he repeated his calculations, and found his hypothesis verified.

In stating the general theory of Induction in the opening chapter, emphasis was laid on the part played by hypotheses or guiding conceptions from the very beginning of an investigation. Frequent references to this point have been made in the discussion of the various methods. We learned that even to define a problem or ask an intelligent question is to *presume* something, or to have some kind of an hypothesis regarding the kind of answer to be given. The question how hypotheses are tested is then really identical with the question how inductions in general are established. Now in explaining and illustrating the procedure of Induction and its use of the various methods, attention was more than once directed to the part played by Elimination. The inductive method of proof, it was said, might be represented by a Disjunctive Syllogism where all the possibilities but one were eliminated by exhibiting their incompatibility with the facts. But in these earlier references it was also indicated that certain qualifications of this view are necessary. It must be borne in mind that Elimination is simply a means

to an end, and that it therefore only partially describes the inductive process. The fact must be emphasized that the real purpose of Induction, as of all thought, is to discover positive connections and laws, and to define these as accurately as possible.

When we observe facts and perform experiments in order to test the first hypothesis suggested by a problem, we obtain evidence which not merely serves to eliminate that hypothesis, but which also points more or less definitely in a positive direction. It is not generally true, then, that we approach a problem with several definite hypotheses in mind, and proceed to try them one after another as we might try various keys at random in a lock. But in thinking, as in all genuine experimentation, failures are instructive. The new hypothesis is forged in and by the process of investigation itself, just as in the progress of the arts finer and more accurate instruments are constantly made possible through the use of those already in existence. The Ptolemaic theory of astronomy, for example, made possible the observations and measurements which finally overthrew it and gave rise to the conception of Copernicus. The new hypothesis, then, may generally be better represented as a modification or closer definition of its predecessor than as something quite new and independent. The formal representation of the Induction by means of the Disjunctive Syllogism, accordingly, fails to bring out clearly the fact of the *development* of knowledge as the work of investigation proceeds. And as a consequence the disjunctive member not eliminated is represented as if it were simply of coördinate importance with the others,

and as if the fact that it was not eliminated were a mere accident. Or put in other words, it fails to make clear the fact that (apart from the unmeaning 'infinite judgment', *e.g.*, 'no good resolution is an octagon') all negation or elimination has positive significance, and that the inductive analysis, as it proceeds, furnishes positive grounds of support for one hypothesis in and through the exclusion of the others. An hypothesis must always be proved by showing its positive conformity with facts: negative results and considerations taken alone never furnish complete inductive proof.

In dealing with certain problems, however, or at certain stages of inquiry, we are often compelled to depend in large part on negative evidence. The fact that other hypotheses are excluded, or are less satisfactory, is very often given as a reason in support of a particular theory. But in such cases there always exist, in addition, positive reasons in support of the theory, though they are not regarded as sufficiently strong to prove it completely. Moreover, at a particular point in an investigation, we are sometimes able definitely to limit the number of possibilities. We do this in mathematics, for example, when we say that one number or dimension is equal to, greater than, or less than, another. And the same is sometimes possible in other fields where we know definitely the exact relations of things. If we are able to say that the phenomenon we are trying to determine is either *a*, *b*, or *c*, we can, of course, prove that it must be *b* by eliminating *a* and *c*. Outside of mathematics, however, the proof would scarcely ever depend wholly on the prin-

ciple of Exhaustion; but in eliminating the other possibilities some positive grounds for the existence of *b* would almost certainly appear.

The method of proving an hypothesis has been described in the following way: If the hypothesis agrees with the facts it is to be regarded as established; if it is not in conformity with them it is to be discarded as false. Now when stated thus baldly, the professed method of proof seems to involve the fallacy of affirming the consequent. 'If a man swallows prussic acid he will die; he is dead, and therefore must have swallowed the acid'. This is obviously fallacious reasoning. We cannot infer that, because certain facts are known to exist which would exist if a certain hypothesis were true, the hypothesis is therefore true. When we speak of an hypothesis as proved by its ability to explain all the facts it is evident that some further qualifications are necessary. From a practical point of view an hypothesis is certain somewhat in proportion to the number and the variety of the facts that it is able to explain, assuming, of course, that there are no important relevant facts which it fails to explain. In speaking of Natural Selection, Darwin says: "This hypothesis may be tested . . . by trying whether it explains several large and independent classes of facts; such as the geological succession of organic beings, their distribution in past and present times, and their mutual affinities and homologies. If the principle of natural selection does explain these and other large bodies of facts it ought to be received." This quotation brings out the fact that the certainty of an hypothesis is not inferred from a

single fact or group of facts, and is even not derived from its agreement with a mere sum of facts. It is rather guaranteed by what has been well called the 'Consilience of Results'. An hypothesis is accepted as established when a number of large and independent bodies of fact all point towards it as the one conception exactly fitted to bring them all into intelligible relations.

From the standpoint of logic it is essential to prove not only that the hypothesis will explain the facts, but that it is the only hypothesis which will explain them. To get this result the other possibilities must obviously be eliminated by a more complete and exact survey of facts, and all the positive circumstances brought to light which tend to confirm the hypothesis in question. This is the function of the 'large and independent bodies of fact' which Darwin mentions in the passage just quoted. What is achieved in this way is the exact fitting together of facts and hypothesis through a process of progressive adjustments. In the process the hypothesis is frequently used as a basis for the prediction of new facts, which, when they are found, serve in their turn to confirm the truth of the hypothesis. A most interesting illustration of this procedure is afforded by Darwin's prediction of the existence of a species of Madagascar moth with a tongue eleven inches in length. The basis of the prediction was his theory of the fertilization of flowers by insects, and the adaptation that is consequently found between the structure of the parts and certain species of insects. Shortly after the appearance of his book *On Fertilization of Orchids by Insects*, a correspondent wrote to

him objecting to the theory elaborated in that work: "What have you to say in regard to an orchid which flourishes here in Madagascar possessing a long nectary, as slender as a knitting-needle, and eleven inches in length? On your hypothesis there must be a moth with a tongue eleven inches long, or this nectary would never have been elaborated." Darwin replied:[1] "The existence of an orchid with a slender nectary eleven inches in length, and with nectar secreted at its tip, is a conclusive demonstration of the existence of a moth with a tongue eleven inches in length, even though no such moth is known." Not long afterwards Darwin's prediction was verified by the discovery of a huge sphinx-moth with a tongue of the length predicted.

§ 75. **Requirements of a Good Hypothesis.** — Various conditions or requisites of a good hypothesis are laid down by writers on logic. The three laws most frequently stated are as follows: (1) That the hypothesis shall be conceivable and not absurd. (2) That it shall be of such a character that deductions can be made from it. (3) That it shall not contradict any of the known laws of nature.

It does not seem to me that the first law is of much value.

[1] I have taken this story from W. H. Gibson's *Blossom Hosts and Insect Guests* (pp. 28–29), but have been unable to verify it from Darwin's published letters. In the second edition of the *Fertilization of Orchids* (Ch. VI), however, Darwin refers to this orchid (*Angræcum sesquipedale*), and from the length of its nectary predicts the existence of a moth with a proboscis of corresponding length. In the same passage he goes on to say: "This belief of mine has been ridiculed by some entomologists, but we now know from Franz Müller that there is a sphinx-moth in South Brazil which has a proboscis of nearly sufficient length, for when dried, it was between ten and eleven inches long. When not protruded, it is coiled up into a spiral of at least twenty windings" (p. 163).

It is largely individual taste or education which leads us to pronounce certain theories 'absurd' or 'inconceivable'. Thus for a long time it seemed inconceivable that the earth should be round, and should revolve on its own axis; and less than a generation ago the theory of evolution, as propounded by Darwin, seemed to many persons 'utterly absurd'. Nor can the third law always be applied as a test of an hypothesis, for many great discoveries seemed, at the time when they were announced, to contradict known laws of nature. The difficulty is that no one is able to affirm unconditionally that a law of nature forbids us to make this or that hypothesis. Of course we feel that a theory is very probably false which is at variance with the law of gravity, or with that of the conservation of energy, or any of the laws which we regard as established beyond a reasonable doubt. But although the chances are always very greatly against any theory which runs counter to what are regarded as well-established laws, there is yet always a *possibility* that it may be true. There is no law of nature so certain as to be absolutely infallible just as it stands. Even those laws which appear to be beyond the possibility of doubt may require to be modified or supplemented. We may find that, practically, it is not wise to trouble ourselves with theories which undertake to overthrow the law of gravitation, or to disprove other fundamental laws of the physical world. But theoretically, at least, there is always a chance — in cases such as we have been supposing the chance is almost infinitely small — that the new theory may be right, and the old one wrong. The practical objec-

tion to admitting the claims of this canon is the difficulty in applying it fairly. The phrase, 'contrary to the laws of nature', like 'inconceivable', and 'absurd', is likely to be used to condemn any theory with which one disagrees. In this way it is evident that the very point is begged which is really at issue.

Of these three canons, therefore, the second appears to state the only condition which is essential to an hypothesis. An hypothesis, if it is to be of any value, must be capable of being proved or refuted. But unless its consequences can be shown by way of deduction, it is impossible to know whether it agrees, or does not agree, with the facts which it is supposed to explain. An hypothesis from which nothing can be deduced, then, is of no value whatever. It always remains at the stage of mere possibility, and without any real connection with fact. It is a mere guess having no significance whatever, for it is entirely incapable either of proof or of disproof. The ability of an hypothesis to lead to the prediction of facts not previously known to exist has sometimes been emphasized as a test of its value. But this circumstance, although making the hypothesis more impressive, is not in itself a proof of its validity. Indeed, true predictions have frequently been made on the basis of hypotheses which were afterwards found incorrect. The essential requirement, however, is that something shall be deducible from the hypothesis, that it shall lead somewhere, and thus afford a programme for further investigation.

In general it is possible to deduce the consequences of a theory only when the principle employed is *analogous*, in

mode of operation, to something with which we are familiar. Thus for example it is because the ether is conceived as resembling other material bodies in important respects that it can be used as a principle of explanation. It is assumed to be elastic and capable of receiving and transmitting vibrations, and as spread out like other material bodies in space. In virtue of these similarities to other material substances it is possible to deduce the consequences which such a substance as ether would imply, and to compare them with the actual facts. But if one should make the assumption that certain phenomena are due to some agency totally unlike anything of which we have any experience, a disembodied spirit, or ghost, for example, it would be impossible either to prove or to disprove the assertion. Knowing nothing whatever of the way in which disembodied spirits act, one could not say whether the phenomena to be explained, table-rapping, planchette-writing, etc., were or were not consistent with a spirit's nature and habits.

Another example of a barren hypothesis from which no conclusions can be drawn is afforded by the 'catastrophe' or 'convulsion' theory in geology, which was first combated by Lyell, in his *Principles of Geology*, published in 1830.

> People had so long held the belief that our earth had only existed a few thousand years, that when geologists began to find a great number of strange plants and animals buried in the earth's crust, immense thicknesses of rock laid down by water, and whole mountain masses which must have been poured out by volcanoes, they could not believe that this

> had been done gradually, and only in parts of the world at a time, as the Nile and the Ganges are now carrying down earth to the sea, and Vesuvius, Etna, and Hecla are pouring out lava a few feet thick every year. They still imagined that in past ages there must have been mighty convulsions from time to time, vast floods swallowing up plants and animals several times since the world was made, violent earthquakes and outbursts from volcanoes shaking the whole of Europe, forcing up mountains, and breaking open valleys. It seemed to them that in those times when the face of the earth was carved out into the mountains and valleys, tablelands and deserts, and when the rocks were broken, tilted up, and bent, things must have been very different from what they are now. And so they made imaginary pictures of how nature had worked, instead of reasoning from what they could see happening around them.[1]

The convulsions or catastrophes thus assumed to take place were regarded as the result of strange incalculable forces whose mode of operation could never be exactly determined. Instead of these mysterious agencies, Lyell assumed that causes similar to those with which we are now acquainted had been acting uniformly for long ages. The nature of the causes at work being known, it became possible to calculate the nature of the effects, and thus to reduce the facts of geology to order and system. As we have already shown, hypotheses which are to prove really serviceable are formed by extending some known principle through analogy to a new class of facts. The assumption

[1] Buckley, *Short History of Natural Science*, pp. 441, 442.

of mysterious agencies and principles whose mode of operation is unlike anything which is known to us does not aid in the extension of knowledge.

EXERCISES (XVIII)

1. Distinguish the terms, fact, law, principle, hypothesis, theory, axiom, postulate.

2. Trace the essential steps for discovering universal laws through induction, showing when and how each of the methods outlined in the preceding chapters play their several parts in attaining the final result.

3. Discuss the two following statements of Darwin: "Any fool can generalize and speculate", and, "No one can be a good observer unless he is an active theorizer."

4. What part does elimination play in the proof of an hypothesis? Explain the nature of the formal fallacy involved in the statement that an hypothesis is established when its results are shown to be true. How is this difficulty overcome?

5. What is the difference between causally and hypothetically connected phenomena?

6. Are hypotheses the same as 'guesses'?

7. What did Newton mean by the famous remark, *hypotheses non fingo* (I frame no hypotheses)?

8. What two methods may be employed to test an hypothesis?

9. What do we mean by an *ad hoc* hypothesis?

CHAPTER XIX

FALLACIES OF INDUCTION

§ 76. **The Source of Fallacy.** — It is necessary at the close of our discussion of the inductive methods to say something regarding the errors to which we are most subject in this kind of thinking. We have seen that knowledge is the result of the mind's own activity and that it grows in completeness through a persistent effort to keep distinct things which are different and to connect phenomena which belong together. Truth, in other words, is gained by intellectual activity. And on the other hand we fall into error, and are led away by false arguments, as a result of mental indolence. Thinking is hard work, and there is always a tendency to avoid it. As a matter of fact we all think much less frequently than we suppose. Usually we are content to follow familiar associations, and to repeat current phrases, without doing any real intellectual work. The difficulty is that we can get along comfortably without thinking for the most part — more comfortably, perhaps, than when we do think. Then again, the mind is less directly under control of the will than the body. One may force himself to sit down at his desk and open a book; but it is more difficult to compel oneself to think.

The only way in which we can be saved from becoming 'intellectual dead-beats' is by the formation of good mental

habits. It requires eternal vigilance and unceasing strenuousness to prevent our degeneration into mere associative machines. What the logical doctrine of fallacies can do is to put us on our guard against this tendency. It enumerates and calls attention to some of the commonest and most dangerous results of slovenly thinking, in the hope that the student may learn to avoid these errors. Some of the fallacies of which we shall treat in this chapter apply equally to deductive or syllogistic reasoning, and have been already treated in Chapter XII. We shall, however, enumerate them here again for the sake of completeness. It is convenient to discuss the various fallacies under the following heads: —

(1) Fallacies due to the careless use of Language.
(2) Errors of Observation.
(3) Mistakes in Reasoning
(4) Fallacies due to Individual Prepossessions.

After what has been said in the preceding chapters regarding the relation of 'facts' and 'theories' it will not be supposed that the distinction between 'errors of Observation' and 'mistakes in Reasoning' is fixed and absolute. Errors in observation result frequently, as we have seen, from inadequate or confused conceptions. There is, however, a relative difference between the two functions of knowledge, which serves as a convenient principle of classification.

§ 77. **Fallacies Due to the Careless Use of Language.** — The careless and unreflective use of words is a very frequent source of error. Words are the signs or symbols of ideas;

but the natural sluggishness of the mind leads often to a substitution of the word for the idea. It is much easier to deal with counters than with realities. Since we must use words to express our thoughts it is almost impossible to prevent them from becoming our masters. Bacon, who gives the name of "Idols of the Market-Place" (*Idola fori*) to the fallacies arising through the use of words, puts the matter in the following striking sentence: "Men imagine that their reason governs words whilst, in fact, words react upon the understanding; and this has rendered philosophy and the sciences sophistical and inactive." [1] The dangers connected with the use of words have also been well represented by Locke, from whom I quote the following passage: —

> Men having been accustomed from their cradles to learn words which are easily got and retained, before they knew or had framed the complex ideas to which they were annexed, or which were to be found in the things they were thought to stand for, they usually continue to do so all their lives; and, without taking the pains necessary to settle in their minds determined ideas, they use their words for such unsteady and confused notions as they have, contenting themselves with the same words other people use, as if their very sound necessarily carried with it constantly the same meaning. . . . This inconsistency in men's words when they come to reason concerning either their tenets or interest, manifestly fills their discourse with abundance of empty, unintelligible noise and jargon, especially in moral matters, where the words, for the

[1] Bacon, *Novum Organum*, Aph. LIX.

most part, standing for arbitrary and numerous collections of ideas not regularly and permanently united in nature, their bare sounds are often only thought on, or at least very obscure and uncertain notions annexed to them. Men take the words they find in use amongst their neighbours; and, that they may not seem ignorant what they stand for, use them confidently, without much troubling their heads about a certain fixed meaning; whereby, besides the ease of it, they obtain this advantage: That, as in such discourses they seldom are in the right, so they are as seldom to be convinced that they are in the wrong; it being all one to go about to draw those men out of their mistakes who have no settled notions, as to dispossess a vagrant of his habitation who has no settled abode.[1]

(1) In treating of the misuse of words, we mention, in the first place, errors arising from the use of a word or phrase in more than one sense. This has already been described as the fallacy of Equivocation. In some cases the equivocation may be mere wilful quibbling on the part of the person propounding the argument, as in the following example of Jevons: —

All criminal actions ought to be punished by law,
Prosecutions for theft are criminal actions,
Therefore prosecutions for theft ought to be punished by law.

Examples of this kind do not mislead any one; but in some instances the change of meaning in words may not be perceived, even by the person who employs the argument. For example, one might reason: —

[1] *Essay Concerning Human Understanding*, Bk. III, Ch. X.

It is right to do good to others,
To assist A in obtaining office is to do him good,
Therefore it is right to assist him in this way.

Here the phrase which is used equivocally is, 'to do good', as will at once be perceived.

(2) Another frequent source of error in the use of words is found in what has been excellently named the Question-begging Epithet. As is well known, there is much in a name. The name may beg the question directly in the terms which it applies, or it may arouse misleading associations. Epithets like 'class-legislation', 'compromise measure', 'a dangerous and immoral doctrine', are terms freely used to describe the measures or views of opponents. And as it is always easier to adopt a current phrase than to examine the facts and draw our own conclusions, it is not surprising that the name settles the whole matter in the minds of so many people. Of course the epithet employed may beg the question in favor of the subject it is used to describe, as well as against it. Politicians well understand the importance of adopting an impressive and sonorous election cry to represent the plank of their party. Thus party cries like 'honest money', 'prohibition and prosperity', 'the people's cause', etc.; are essentially question-begging epithets. Even words like 'liberty', 'justice', and 'patriotism', are frequently used in such a way as to bring them under the class of fallacies which we have here described. Under this heading, also, may be grouped 'cant' words and phrases. When we accuse a person of using cant, we always imply that he is more or

less consciously insincere, that he is professing opinions and sentiments which he does not really possess. Any insincere expression which is made primarily for the sake of effect may be rightly termed cant. It is not even necessary that the speaker should be fully conscious of his insincerity. A man may easily deceive himself, and, as he repeats familiar words and phrases, imagine himself to be overflowing with patriotism, or with sympathy for others, or with religious feelings.

(3) Figurative Language is another frequent source of error. Of the various figures of speech perhaps metaphors are the most misleading. The imagery aroused by metaphorical language is usually so strong as to make us forget the difference between the real subject under consideration and the matter which has been used to illustrate it. Thus in discussing problems of mind it is very common to employ metaphors drawn from the physical sciences. For example we read in works on psychology and ethics of 'struggle of ideas', of 'the balancing and equilibration of motives', of 'action in the *direction* of the strongest motive', etc. Another illustration which has been often quoted is Carlyle's argument against representative government, founded on the analogy between the ruler of a state and the captain of a ship. The captain, he says, could never bring the ship to port if it were necessary for him to call the crew together to get a vote every time he wished to change the course. The real difference between the relation of a captain to his crew, and the executive officers in a state to the citizens, is lost sight of by the metaphor.

Metaphors should be used only to illustrate and suggest and never to prove. Metaphorical reasoning is simply a case of analogy, the imperfections and dangers of which have been already pointed out. It is however one of the errors most difficult to avoid. A hidden metaphor lurks unsuspected in many of the words in common use. We may thus appreciate the force of Heine's humorous petition: "May Heaven deliver us from the Evil One, and from metaphors." It is of course not necessary or desirable to abstain entirely from the use of metaphors. What is essential is to prevent them from 'reacting upon the understanding'. A person who is able to employ many metaphors drawn from various fields is perhaps less likely to be misled by them, than the unimaginative man — the man of one figure and one phrase — whose mind sticks in mechanical grooves.

§ 78. **Errors of Observation.** — Sometimes insufficient observation is the result of a previously conceived theory; sometimes it may be due to inattention, to the difficulties of the case, or to lack of the proper instruments and aids to observation. We have already had occasion to refer to the influence of a theory on observation (cf. § 57). As a rule we see only those instances which are favorable to the theory or belief which we already possess. It requires a special effort of attention to take account of negative instances and to discover the falsity involved in some long-standing belief. Indeed it requires quite as much mental alertness to overthrow an old theory as to establish a new one. It is obvious that the fallacy here is due, as is generally

the case, to insufficient observation and analysis. The conclusion is based on an uncritical use of the method of Agreement, without any attempt to compare the positive case with instances where the phenomenon is absent. This comparison is made by the method of Difference. This tendency of the mind to seize upon affirmative instances, and to neglect the evidence afforded by negative cases, is well set forth by Bacon in the following passage: —

> The human understanding, when any proposition has been once laid down (either from general admission and belief, or from the pleasure it affords), forces everything else to add fresh support and confirmation; and although most cogent and abundant instances may exist to the contrary, yet either does not observe or despises them, or gets rid of and rejects them by some distinction, with violent and injurious prejudice, rather than sacrifice the authority of its first conclusions. It was well answered by him who was shown in a temple the votive tablets suspended by such as had escaped the peril of shipwreck, and was pressed as to whether he would then recognize the power of the gods; 'But where are the portraits of those who have perished in spite of their vows?' All superstition is much the same, whether it be that of astrology, dreams, omens, retributive judgment, or the like, in all of which the deluded observers observe events which are fulfilled, but neglect and pass over their failure, though it be much more common. But this evil insinuates itself still more craftily in philosophy and the sciences, in which a settled maxim vitiates and governs every other circumstance, though the latter be much more worthy of confidence. Besides, even in the absence of that eagerness and want of thought (which

we have mentioned), it is the peculiar and perpetual error of the human understanding to be more moved and excited by affirmatives than negatives, whereas it ought duly and regularly to be impartial; nay, in establishing any true axiom the negative instance is the most powerful.[1]

The nature of this fallacy has been so well illustrated by the quotation just given that we may pass on at once to speak of other cases of insufficient observation. Our discussion of the processes of reasoning has made it clear how necessary it is to observe carefully and attentively. The majority of the false theories that have appeared in science and in philosophy, as well as those of common life, have arisen from lack of observation. The doctrine of innate ideas, and the theory that combustion was a process of giving off phlogiston — a substance supposed to be contained in certain bodies — may be given as examples. With regard to phlogiston, Mill says:—

> The hypothesis accorded tolerably well with superficial appearances: the ascent of flame naturally suggests the escape of a substance; and the visible residuum of ashes, in bulk and weight, generally falls extremely short of the combustible material. The error was non-observation of an important portion of the actual residue; namely, the gaseous products of combustion. When these were at last noticed and brought into account, it appeared to be a universal law that all substances gain instead of losing weight by combustion; and after the usual attempt to accommodate the old theory to the new fact by means of an arbitrary hypothesis (that

[1] *Novum Organum*, Bk. I, Aph. XLVI.

phlogiston had the quality of positive levity instead of gravity), chemists were conducted to the true explanation, namely, that instead of a substance separated, there was, on the contrary, a substance absorbed.[1]

This illustration also exemplifies the consequences both of neglecting Residues and of noticing and seeking to explain them. In some seaside communities there is a belief that living beings, both human and animal, never die at flood tide. 'They always go out with the ebb', it is said. Again, there is a general belief, shared by such an eminent scientist as Herschel, that the full moon in rising possesses some power of dispersing the clouds. Careful observations made at the Greenwich observatory have however shown conclusively that the moon has no such power as that supposed.[2]

Another circumstance to be considered in this connection is the inaccuracy and fallibility of ordinary memory. Every one must have noticed how rarely two persons agree completely in the reports they give of a conversation they have heard, or of events they have experienced. This is due in part to diversity of interest; each person remembers those circumstances in which for any reason he is most strongly interested. But in addition, it is largely the result of the inevitable tendency of the mind to confuse what is actually observed with inferences made from its observations. The inability to distinguish between what is really perceived, and what is inferred, is most strongly marked in uneducated

[1] *Logic*, Bk. V, Ch. IV.
[2] Cf. Jevons, *Principles of Science*, Ch. XVIII.

persons, who are not on their guard against this fallacy. An uneducated person is certain to relate, not what he actually saw or heard, but the impression which the events experienced made upon him. He therefore mixes up the facts perceived with his own conclusions drawn from them and with statements of his own feelings in the circumstances. A lawyer who has to cross-examine a witness is usually well aware of this tendency and may take advantage of it to discredit the testimony. The experienced physician knows how worthless is the description of symptoms given by the ordinary patient, or by sympathetic friends, or by an inexperienced nurse. The more one's sympathies and interests are aroused in such a case, the more difficult it is to limit oneself to an exact statement of actual occurrences.

But this tendency is not confined to persons deficient in knowledge and ordinary culture. It usually requires special training to make one a good observer in any particular field. It is by no means so easy as it may appear to describe exactly what one has seen in an experiment. If we know, or think that we know, the explanation of the fact, there is an almost inevitable tendency to substitute this interpretation for the account of what has been actually observed. Recent psychological investigation, aided by exact experimental methods, has done much to disentangle the data of perception from inferences regarding these data. As every one knows who has practised psychological introspection, it is only with the utmost difficulty, and after long training, that one can distinguish the actual psychological processes present to consciousness, from the associative and logical elements

which are bound up with them in our ordinary experience. The following passage from Mill deals with this question: —

> The universality of the confusion between perceptions and the inferences drawn from them, and the rarity of the power to discriminate the one from the other, ceases to surprise us when we consider that in the far greater number of instances the actual perceptions of our senses are of no importance or interest to us except as marks from which we infer something beyond them. It is not the color and superficial extension perceived by the eye that are important to us, but the object of which these visible appearances testify the presence; and where the sensation itself is indifferent, as it generally is, we have no motive to attend particularly to it, but acquire a habit of passing it over without distinct consciousness, and going on at once to the inference. So that to know what the sensation actually was is a study in itself, to which painters, for example, have to train themselves by long-continued study and application. In things further removed from the dominion of the outward senses, no one who has not had great experience in psychological analysis is competent to break this intense association; and when such analytic habits do not exist in the requisite degree, it is hardly possible to mention any of the habitual judgments of mankind on subjects of a high degree of abstraction, from the being of God and the immortality of the soul down to the multiplication table, which are not, or have not been, considered as matter of direct intuition.[1]

In pointing out the evils arising from confusing fact and theory, it is not forgotten that what are taken as 'facts'

[1] *Logic*, Bk. V, Ch. IV, § 5.

are the results of earlier theorizings and interpretations (cf. § 57). But the results of past processes of combination and comparison become embodied or fixed in more or less definite form in the course of experience. Moreover they are fixed in language — whether in the language of common life or in the technical terminology of the different sciences. There always is a kind of convention conveyed, both by the language of ordinary life and by that of the sciences as to what may be taken as a fact in that court circle — *i.e.*, taken for granted as a datum or starting-point for further construction. What is a fact in science may of course be an inference from the standpoint of popular knowledge, or *vice versa.*

Now the fallacy against which warning is here given arises from not understanding clearly what, in any given circumstance, may properly be taken as 'fact'. If there is confusion as to the starting-point there is no proper basis on which to construct a theory. Moreover, without some certain starting-point, some well-ascertained datum, there is no means of testing and criticising our theories.

§ 79. **Mistakes in Reasoning.** — The problem of the inductive processes of reasoning is to ascertain what facts are necessarily and essentially connected, and to explain this connection. Now in order to distinguish between chance conjunctions of phenomena and real causal connections, careful and extensive observation, aided whenever possible by experiment, must be employed. In short, to establish a real law of connection between phenomena it is necessary to use one or more of the inductive methods described in Chapter XVI. But to do this implies, in many cases, long

processes of analysis; the performance of intellectual work, which ordinary minds, at least, have the tendency to shirk whenever possible. It is much easier to allow associations to control our thoughts, and to assume, (1) that events which happen together in our experience a number of times are causally connected; or, (2) that things that are in some way alike are causally connected, or of the same kind. We are led to such a conclusion by a natural psychological tendency, without taking any thought about the matter, while logical analysis and discrimination require a distinct conscious effort.

The general name used to describe the first class of fallacies due to this particular form of mental sluggishness is *post hoc, ergo propter hoc.* Two events occur in close conjunction with each other and it is then assumed without further investigation that they are related to each other as cause and effect. Many popular superstitions are examples of this fallacy. Some project begun on Friday turns out disastrously, and it is inferred that some causal relation existed between the fate of the enterprise and the day on which it was begun. Or thirteen persons sit down to dinner together and some one dies before the year is out. It is to be noticed that such beliefs are supported by the tendency referred to in the last section, to observe only the instances in which the supposed effect follows, and to neglect the negative cases, or cases of failure. 'Fortune favors fools', we exclaim when we hear of any piece of good luck happening to any one not noted for his wisdom. But we fail to take account of the more usual fate of the weak-minded. The belief that

the full moon in rising disperses the clouds, which was also quoted earlier, is a good example of *post hoc, ergo propter hoc*. In fact all the fallacies treated in this chapter, except those due to language, might quite properly be included under this heading. The tendency to neglect negative instances was given by Bacon as the most striking example of the "Idols of the Tribe" (*Idola tribus*), *i.e.*, of the species of fallacies to which the whole tribe or race of men are subject.

A special case of this fallacy, to which attention may be called separately, arises from hasty generalization, or generalization on an insufficient basis of fact. The term 'generalization' is often used in logic to denote the whole inductive movement of thought from particular facts to general principles and laws. But the fallacy to which reference is here made usually concerns a special stage in that process — the stage where a first generalization is made from instances. We are said to generalize when, after a more or less extended and careful set of observations, we take the instances observed as typical of all phenomena of the same field, or of the same general character. When due care has not been exercised in making the observations, or when the observations are few in number, or all drawn from a limited part of the whole field, we speak of 'hasty generalization'. Thus it is not unusual to hear a traveler declare, on the basis of a very limited experience, that 'the hotels of some city or country are thoroughly bad'. The generalizations so frequently made regarding the peculiar characteristics of Americans, or Englishmen, or

Frenchmen are usually of the same sort. What is exceptional tends to attract the attention more than what is usual and normal; hence the tendency *to take the exceptional for the typical*. Even scientific books are not always free from this error. In a recently published psychological study of the first year of the life of a child, by the mother, it was explained why a baby always sucks its thumb rather than its fingers. The explanation was that the thumb, being on the outside and projecting outwards, got oftenest into the baby's mouth, and so the habit was formed. The mother assumed what she had observed in her own child to be true universally. Other parents declare that their babies never put the thumb into the mouth, but always the fingers or the whole hand.

Another fallacy belonging to this group arises from the uncritical use of Analogy. False Analogy is closely connected with the fallacies of figurative language. Indeed, the latter type of fallacies, almost without exception, arise from a loose use of Analogy. It has been pointed out (§ 71), that the value of an inference from Analogy depends upon the 'depth' or 'importance' of the resemblances upon which it is based. False inferences arise in every field from taking some striking or surface resemblance as the basis of a conclusion. Nothing is easier than to be led uncritically by vague resemblances, or even to imagine them where they do not exist. Vague or fancied analogies are the foundation of many popular superstitions regarding omens, illness, cures, etc., and also play an important part in many of the sympathetic and imitative practices of Magic.

§ 80. **Fallacies Due to Individual Prepossessions.**— Bacon named this class of fallacy "The Idols of the Cave". Each individual, as he represents the matter, is shut up in his own cave or den; that is, he judges of things from his own individual point of view. In the first place, one's inclinations and passions, likes and dislikes, pervert one's judgment. It is exceedingly difficult, as we all know, to be fair to a person we dislike, or to refrain from judging too leniently the shortcomings of those to whom we are warmly attached. Again, it is not easy to put oneself in the position of an impartial spectator when one's interests are at stake. "The understanding of men", says Bacon, "resembles not a dry light, but admits some tincture of the passions and will". Furthermore, each individual has a certain personal bias as a result of his natural disposition and previous training. Thus it is almost impossible for an individual to free himself from national prejudices, or from the standpoint of the political party, or the church in which he was brought up. Or if a person does give up his old views, he not infrequently is carried to the opposite extreme, and can see no good in what he formerly believed. Even education and the pursuit of special lines of investigation may beget prejudices in favor of particular subjects. When a man has been engaged exclusively for a long time in a particular field, employing a particular set of conceptions, it is almost inevitable that he should look at everything with which he has to do in the same light. The mathematician's view of the world is almost sure to be different from that of the historian, or that of the student of æsthetics. It is very

difficult for the physicist to conceive of any natural process except in terms of molecules and vibrations. It is inevitable that each man should be blinded to some extent by his own presuppositions. But to recognize one's limitations in this respect, is to pass, to some extent at least, beyond them.

Moreover, each age, as well as each individual, may be regarded as governed largely by current presuppositions and prejudices. Bacon does not however classify the errors into which one may be led by the spirit of the time (*Zeitgeist*), or the beliefs derived from the past, with the "Idols of the Cave", but speaks of them rather as "Idols of the Theatre" (*Idola theatri*). He draws his examples of this from the influence which the traditions of the Schoolmen still continued to exert in his own day. Throughout the Middle Ages theological doctrines and opinions controlled almost absolutely the opinions and beliefs of mankind. This influence, doubtless, still makes itself felt, but people are now pretty generally awake to the dangers from this source. On the other hand it is more difficult to realize at the present time that it is not impossible for prejudices and prepossessions to grow out of scientific work. The success of modern scientific methods has sometimes led investigators to despise and belittle the work of those who do not carry on their investigations in laboratories, or do not weigh and measure everything. But conceptions and methods that prove useful in one science cannot always be employed profitably in another. A conception or mode of regarding things which has proved serviceable in one field is almost certain to dominate a whole age, and to be used as an almost universal

principle of explanation. The eighteenth century, for example, was greatly under the influence of mechanical ideas.[1] Newton's discovery made it possible to regard the world as a great machine, the parts of which were all fitted together according to the laws of mechanics. This view led to such a vast extension of knowledge in the realm of physics and astronomy that the conceptions upon which it is based were applied in every possible field — in psychology, in ethics, in political science. The world itself, as well as religious creeds and political and social institutions, were supposed to have been deliberately made and fashioned by some agent. Again, at the present time we are dominated by the idea of evolution. The biological notion of an organism which grows or develops has been applied in every possible field. We speak, for example, of the world as an organism rather than as a machine, of the state and of society as organic. And the same conception has been found useful in explaining the nature of human intelligence. It is easy for us to realize the limitations and insufficiency of the notion of mechanism as employed by the thinkers of the eighteenth century. But it is not improbable that a future century may be able to see more clearly than we are able to do, the weaknesses and limitations of the conception which has proved so fruitful in this generation.[2]

[1] See Creighton, "Eighteenth and Nineteenth Century Modes of Thought," in *The Philosophical Review*, Vol. XXXV, No. 1, Jan., 1926, pp. 1–21.
[2] See Smart, *The Logic of Science*, Ch. VI and *passim*.

EXERCISES (XIX)

1. What is the source of fallacy? How far is it true that the study of logic can protect us from fallacies?

2. Mal-observation and false analogy are implicitly generalizations which are erroneous (Mellone). Discuss.

3. What fallacy is committed in arguing that day and night are causes of one another?

4. In how many ways may the observation of facts be fallacious?

5. In what form are the several fallacies described by Bacon likely to appear at the present day?

6. Compare the fallacies of false analogy and (*a*) of figurative language, (*b*) of negative instances.

PART III

THE NATURE OF THOUGHT

CHAPTER XX

JUDGMENT AS THE ELEMENTARY PROCESS OF THOUGHT

§ 81. **Thinking the Process by Which Knowledge Grows or Develops.** — Logic was defined as the science of thinking, and we have seen that the business of thought is to furnish the mind with truth or knowledge. Under what general conception, now, shall we bring thinking, and what method shall we adopt to aid us in its investigation? It is at once clear that the conscious process by which knowledge is built up does not resemble mechanical processes like pressure, or attraction and repulsion. It is more nearly related to something living, like a plant or an animal, which grows or develops from within, in accordance with the laws of its own nature. In short, thinking must be regarded as a process rather than as a dead thing.

When thought is regarded in this way, moreover, a method of procedure at once suggests itself. In these days we are very familiar with the notion of evolution or development, and the application of this notion has proved of the greatest service to science, and particularly to those sciences dealing with the phenomena of life. What is characteristic of this manner of regarding things is the fact that it does not consider the various phenomena with which it deals as fixed, unchangeable things, each with a ready-made nature of its own. But each thing is simply a stage

of a process, a step on the way to something else. And the relations of the various phenomena to each other, their connection and unity as parts of the one process, come out more clearly when viewed in this way. In other words, by taking a survey of the genesis and growth of things, or the way in which they come to be, we gain a truer idea of their nature and relations than would be possible in any other way. The past history of any phenomenon, the story of how it came to be what it is, is of the greatest possible service in throwing light upon its real nature. Now one cannot doubt that this conception will also prove serviceable in the study of logic. That is to say, it will assist us in gaining a clearer idea of the nature of thinking to conceive it as a conscious function, or mode of acting, which unfolds or develops in accordance with the general laws of organic evolution. And this process may be supposed to go on both in the individual, as his thought develops and his knowledge expands, and in the race, as shown by its history. By adopting this notion we may hope to show also that there is no fundamental difference in kind between the various intellectual operations. Judgment and Inference, for example, will appear as stages in the one intellectual process, and the relation between Induction and Deduction, as each having its own work to do, will become evident.

§ 82. **The Law of Evolution and Its Application to Logic.** — The most striking characteristic of any organism at a low stage of development is its almost complete lack of structure. An amœba, for example, can scarcely be said

to have any structure; it is composed of protoplasm which is almost homogeneous, or of the same character throughout. When however we compare an amœba with an animal much higher in the scale of life, *e.g.*, a vertebrate, a great difference is at once evident. Instead of the simple, homogeneous protoplasm, the organism is composed of parts which are unlike, or heterogeneous, such as bones, muscles, tendons, nerves, blood-vessels, etc. The process of evolution from the lower organism to the higher has brought with it a differentiation of structure. That is, in the amœba there are no special organs of sight, or hearing, or digestion, but all of these acts seem to be performed by any part of the organism indifferently. In the vertebrate, on the other hand, there is division of labor, and a separate organ for each of these functions. One may also notice that the same change is observable when the *acts or functions* performed by a lower organism are compared with those of a higher. The life of the amœba seems to be limited almost entirely to assimilation and reproduction; but when we advance from the lower animals to the higher, and from the higher animals to man, there is an ever-increasing complexity and diversity in the character of the actions performed. We thus see how the process of evolution involves differentiation both of *structure* and of *function*, in passing from the homogeneous to the heterogeneous.

But differentiation, or increase in diversity, is only one side of the process of evolution. As we pass from a lower to a higher stage the various parts of an organism are seen to become more essential to one another. If certain plants

or low animal organisms are divided into several parts each part will go on living. Its connection with the other parts does not seem to have been at all necessary to it. But when we are dealing with higher forms of life each part is seen to have its own particular function, and to be essential both to the other parts and to the organism as a whole. In other words the parts now become members, and the whole is not simply an aggregation of parts or pieces, but is constituted by the necessary relation of the members to one another. The more highly evolved the whole with which we are dealing, the more closely connected and essential to one another are the various parts seen to be. It becomes increasingly true that if one member suffers all the other members suffer along with it. The same principle is illustrated by the relation of classes and individuals in modern society. In spite of the conflicts between capital and labor, between rich and poor, it is becoming increasingly evident that the unity of society is more fundamental than its differences and antagonisms.

Evolution, then, not only exhibits a constant process of differentiation and a constant increase in the diversity of parts and organs, but there goes along with this what might be called a process of unification whereby the parts are brought into ever closer and more essential relation to one another. In this way a *real or organic whole*, as opposed to a mere *aggregate*, is formed.

The application of this general law of evolution to the development of the thinking process is not difficult. We shall expect to find that thinking, in its first beginnings,

both in the individual and in the race, will be much less complex and differentiated than at a higher stage. That is, the earliest or simplest thinking tends to take things in a lump, without making any distinctions. The infant, for example, does not distinguish one person from another, or perhaps does not distinguish even the parts of its own body from surrounding objects. Now it is clear that intellectual development, growth in knowledge, must in the first place involve differentiation. What is complex must be analyzed or separated into its various parts. Things which are different must be distinguished and clearly marked off from one another. The development of thought implies, then, as one of its moments, discrimination or analysis — what we previously called differentiation.

The other moment of the law of evolution, integration, also finds a place in the development of thought, and goes hand in hand with the former. The child and the uneducated man not only often fail to make distinctions where these really exist, but the parts of their knowledge are fragmentary and have little or no relation to one another. The various pieces of their knowledge are like the parts of the amœba — they may be increased or diminished without themselves undergoing any change. But in order to pass from a lower to a higher intellectual point of view — to become better educated, in a word — it is necessary to see the way in which the various pieces of our knowledge are connected and dependent upon one another. It is not enough to analyze and keep separate things which are distinct, but it is also necessary to understand how the

various parts of our knowledge are inter-related and essentially dependent on one another. In other words we may say that it is characteristic of our intelligence to endeavor to put things together so as to form a whole, or system, of interconnected parts. And the more completely it is able to do this (provided that the process of differentiation has also made a corresponding advance), the higher is the stage of development which has been attained. The ideal of knowledge, or of complete intellectual development, would be to understand the oneness and relation of everything that exists, even of all those things which seem now to be entirely different in kind. A knowledge of any one fact would then carry with it a knowledge of every other fact. Or rather, our knowledge would be so completely unified that each part would show the nature of the whole or system to which it belongs; just as a leaf of a plant, or a tooth of an animal, may be sufficient to tell the naturalist of the wholes to which they belong.

This of course will always remain an ideal; but it is in this direction that thinking actually develops. It is a step in advance to discover the reasons for any fact which one previously knew as a mere fact. For to discover the reasons for a fact is to bring it into connection with other facts, to see them no longer as isolated and independent, but as belonging together to one group or system of facts. And the further the process of explanation goes on, the more completely is our knowledge unified and related.

There is another fact implied in the very nature of evolution, of which logic, as well as the other sciences, may take

advantage. We have assumed that the more complete and difficult kinds of thinking have grown or developed from simpler types of the same process, and not from something different in kind. It will therefore follow that the essential characteristics of the thinking-process may be discovered in its simplest and most elementary form. It is found that all the essential functions of the fully developed organism are discharged by the primitive cell. And because it is easier to study what is simple than what is complex, the cell is taken as the starting-point in biology. Similarly there will be an advantage in beginning with the simplest and most elementary forms of thinking. What is found true of these simple types of thought may be assumed to be essential to the thinking-process as such.

§ 83. **Judgment as the Starting-point.** — What, then, is the simplest form of thinking? What shall we take as a starting-point, corresponding to the cell in biology, or the elementary process in psychology? To answer this question it is not necessary first to decide where in the scale of animal life thinking actually begins. We shall not be obliged to discuss the much-debated question, whether or not dogs think. Wherever thinking may be found it is essentially an activity of the mind. When it is present, that is, there is always intellectual work done, something interpreted or put together, and a conclusion reached. One may perhaps say that thinking is simply the way in which the mind puts two and two together and sees what the result is. It implies that the mind has waked up to the significance of things and has interpreted them for itself. Suppose

that one were sitting in one's room very much engaged with some study, or wrapped up in an interesting book, and suppose that at the same time the sound of a drum should fall upon one's ears. Now the sound sensations might be present to consciousness without calling forth any reaction on the part of the mind. We might be so intent on our book that we should not wake up, as we have been saying, to the meaning or significance of the drum-taps; or perhaps not even to the fact that they were drum-taps at all. But if the mind did react upon the sound sensations it would try to interpret them, or put them together so as to give them a meaning. As a result some conclusion would be reached, as, for example, 'the drum is beating'; or sufficient intellectual work may have been done to give as a conclusion, 'that is the Salvation Army marching up the street'. In any case it is of the greatest importance to notice that the conclusion does not come into our minds from without, but that it is the product of the mind's own activity, as has been described. It is not true that knowledge passes into our minds through the senses; it is only when the mind wakes up to the meaning of sensations and is able to put them together and interpret them that it gains any knowledge.

Now the simplest form of such an act of thought is called a judgment. Judgment, we may say, is a single intellectual act of the kind we have described; and its conclusion is expressed by means of a Proposition; as, for example, 'the grass is green', 'the band is playing'. In accordance with general usage, however, we may use the

term 'Judgment' for both the act itself and its result. And the word 'Proposition' will then denote the external expression in speech or writing of the product of an act of judgment.

In our investigation of the nature of thought, then, we must begin with Judgment. There are three things we shall have to do: (1) To endeavor to discover the fundamental characteristics of this simple type of thinking; (2) To show the various forms which it assumes, or to describe the different kinds of Judgment; and (3) To trace the process by which Judgment expands into the more complete logical form of Inference. Before any of these questions are considered, however, it is necessary to meet a very serious objection to our whole procedure of beginning with Judgment as the elementary process of thinking.

§ 84. **Concepts and Judgment.** — In the last section we endeavored to show that Judgment is the elementary process of thought, and that with it all knowledge begins. The same position was also maintained in an earlier chapter. This view, however, may seem to be contradicted by the treatment of Judgment usually found in logical textbooks.

Judgment, it is said, is expressed by a proposition; and a proposition is made up of three parts, subject, predicate, and copula. Thus in the proposition 'iron is a metal', 'iron' is the subject, 'a metal' the predicate, and the two terms are joined or united by means of the copula 'is'. A Judgment is therefore defined as an act of joining together, or, in negative judgments, of separating, two concepts or

ideas. If this account be accepted it follows that the ideas composing the judgment (iron and metal, in the example given above) are pieces of knowledge which precede the judgment itself. And the act by which these logical ideas (or, as they are usually called, concepts) are formed must also be earlier and more fundamental than the act of judging. It is therefore held that logic should begin with concepts, the elements out of which judgments are compounded, and that the first logical act consists in the conception or simple apprehension of the ideas or concepts.

It is necessary to examine this position very carefully. What is maintained is that a process of forming concepts, or logical ideas, presumably quite distinct from the activity of judgment, necessarily precedes the latter. Before it is possible to judge that 'iron is a metal', for instance, one must have gained, by means of Conception or Apprehension, the ideas denoted by the subject and predicate of this proposition. Judgments, that is, are made or compounded out of something different from themselves.

It may be well to begin the defence of our own position by noting what is undoubtedly true in what has just been stated. In making a judgment like 'iron is a metal', it is of course necessary to have the concept 'iron' and the concept 'metal'. But what is implied in having a concept of anything? Let us suppose that a person is making the above-mentioned judgment for the first time — that is, really drawing a conclusion for himself and not merely repeating words. He would begin, we may say, with the concept 'iron'. But if this concept is more than a mere

word, if it really means anything, it must have been formed by a number of judgments. The concept 'iron', if it has any significance for the person using it, means a definite way of judging about some substance — that it is hard, malleable, tough, etc. The greater the number of judgments that the concept represents, the more meaning or significance it has; apart from the judgment it is a mere word and not a thought at all.

To admit, then, that in judging we always start from some concept, does not imply that there is a different form of intellectual activity prior to judgment, furnishing the latter with ready-made material for its use. But as we have seen, in ordinary judgments like the example with which we have been dealing, the new judgment is a further expansion or development of a previous set of judgments represented by the concept. The concept, then, stands for the series of judgments already made. Language comes to the aid of thought and makes it possible to gather up such a set of judgments and represent them by a single expression — often by a single word. Every word serving as the name of some logical concept represents intellectual work — the activity of judgment — in its formation. In learning our own language we inherit the word without doing the work. But it must never be forgotten that the word in itself is not the concept. To make the thought our own, to gain the real concept, it is necessary to draw out or realize to ourselves the actual set of judgments for which the word is but the shorthand expression.

The view that regards the judgment as a compound of

two parts — subject and predicate — rests upon the substitution of words for thoughts. It analyzes the *proposition*, (the verbal or written expression of the judgment), instead of the judgment itself. In the simplest types of proposition the parts often do seem to exist independently of each other. The subject usually stands first, and is followed by the predicate.[1] But there is no such order of parts in a judgment. When one judges 'it is raining', or 'that is a drum', the piece of knowledge is one and indivisible. And the act by which this knowledge is gained is not an external process of joining one part to another but is an intellectual reaction by which we recognize that something, not previously understood, has a certain meaning or significance.

Again, it is only when concepts are identified with the words making up the parts of the proposition that they can be regarded as ready-made existences quite independent of their connection in a judgment. The terms 'iron' and 'metal' are separable parts of the proposition and exist independently of their connection with it. The conclusion has been therefore drawn that concepts had a like independence of judgments, but might enter into the latter and form a part of them without affecting their own nature in any way. But as we have already seen, the concept has no meaning apart from the series of judgments it represents. And as thinking goes on, and new judgments are made, its nature is constantly changing. In short, concepts are not dead *things* but living *thoughts* in constant process of development.

[1] But see above, pp. 74 ff.

The objection, then, which urges that conception is a logical process prior to judgment, turns out, when rightly understood, to be no objection at all. For in the light of what has been said, it only amounts to this: In making new judgments regarding anything we must set out from what we already know of it, as represented by the judgments already made. That is, the starting-point for a new judgment is the concept or series of judgments which represents the present state of our knowledge. The progress of knowledge is not from the unknown to the known, but from a state of partial and incomplete knowledge to one of greater perfection. Thus the judgment 'gold is malleable' (supposing it to be a genuine judgment made for the first time) adds to, or develops farther, our existing knowledge of gold, as represented by a series of judgments previously made regarding it.

It may however be urged that not every judgment can grow out of previous judgments in this way. For if we go back far enough we must reach some judgment which is absolutely first, and which presupposes no antecedent judgment. This is like the paradox regarding the origin of life. If all judgments are derived from antecedent judgments, how was it possible for the first one to arise? It will perhaps be sufficient answer to deny the existence of the paradox. Consciousness must be regarded as having from the first the form of a judgment. No matter how far one goes back in the history of consciousness, one will always find, so long as consciousness is present at all, some reaction, however feeble, upon the content, and something like knowl-

edge resulting. Even the consciousness of the newly born infant reacts, or vaguely judges, in this way. These primitive judgments are of course very weak and confused, but they serve as starting-points in the process of intellectual development. Growth in knowledge is simply the process by means of which these vague and inarticulate judgments are developed and transformed into a more complete and coherent experience.

EXERCISES (XX)

1. Review the applications of the idea of evolution or development to the exposition and interpretation of logical doctrines in Parts I and II of this work.

2. Connect this review with our exposition in the present Chapter.

3. What do you understand by Judgment? How does the view of mind which takes judgment as the elementary process differ from that of psychology?

4. How does a psychogenetic study of thought differ from our consideration of it from a developmental point of view?

5. Would the doctrine that in knowing we first have simple apprehension, then, as separate intellectual processes, judgment, and finally inference, agree with the general evolutionary view of consciousness? Explain fully.

CHAPTER XXI

THE MAIN CHARACTERISTICS OF JUDGMENT

§ 85. **The Universality of Judgments.** — We have now to examine the nature of Judgment a little more closely than has been done hitherto. In the first place, we note that all judgments claim **Universality**. There are, however, several kinds of universality, and more than one sense in which a judgment may be said to be universal. We speak of a universal judgment (more properly of a universal proposition) when the subject is a general term, or is qualified by some such word as 'all', or 'the whole'. And we distinguish from it the particular judgment, where the subject is only the part of some whole, and is usually preceded by 'some', or by other partitive words. But here we have no such distinction in mind; we are speaking of the universality belonging to the very nature of Judgment as such, and shared in by judgments of every kind.

When we say that judgments are universal in the sense in which the word is now used, we mean that the conclusions they reach claim to be true for every one. No matter what the subject and the predicate may be, a judgment, *e.g.*, 'man is mortal', comes forward as a fact for all minds. We have shown in the last chapter that it is by judging, or putting things together for itself, that the human mind gains knowledge. Now the assumption upon which this

process is based is that the result thus reached — knowledge — is not something merely individual and momentary in character. When I judge that 'two and two are four', or that 'iron has magnetic properties', the judgment is not merely a statement of what is going on in my individual consciousness; but it claims to express something true for other persons as well as for me. It professes to deal with facts that are true, and in a sense independent of any individual mind. The judgments by which such conclusions are reached are universal, then, in the sense that they are asserted as true for every one and at all times. The word 'objective' has essentially the same meaning. Although each man reaches truth only by actually judging for himself, yet truth is objective, out there beyond his individual or 'subjective' thought, shared in by all rational beings. The assumption upon which all argument proceeds is that there is an objective standard and that if people can be made to think they will arrive at it. Thought is in essence a process of self-criticism; for it has in itself its own standard of truth, which comes to light in and through the process of development.

The only alternative to this position is scepticism, or pure individualism. If Judgment is not universal in the sense that it reaches propositions true for everybody, it is of course impossible to find any standard of truth at all. The judgments of any individual in that case would simply have reference to what seemed true to him at the moment, but could not be taken to represent any fixed or permanent truth. Indeed if one regards Judgment as dealing merely

with particular processes in an individual mind the ordinary meanings of a truth and falsehood are completely lost, and it becomes necessary to give a new definition of the words. This was the position of the Sophists at the time of Socrates. Each individual man was declared to be the measure of what is true and false, as well as of what is good and bad. There is thus no other standard of truth or value than the momentary judgment (or caprice) of the individual. This is, in a way, the *reductio ad absurdum* of scepticism.

The common nature of truth as something in which all can share, presupposes, then, a common mode of thinking or judging on the part of all rational beings. And it is this universal type or form of knowing with which logic deals. The question as to whose thought is investigated, or in what individual mind the thought takes place, is in itself of no importance. The consciousness of a savage differs very greatly from that of an educated man; it is much less complex and less highly developed. But in spite of such enormous differences there exists in both an intelligence, or way of thinking, showing the same essential character and operating according to the same fundamental laws.

§ 86. **The Necessity of Judgments.** — The second characteristic belonging to Judgment is **Necessity**. By this we mean that when a person judges he is not free to reach this or that conclusion at will. As an intellectual being he feels bound to judge in a certain way. This is sometimes expressed by saying that we cannot believe what we choose; we must believe what we can.

In many of the ordinary judgments of everyday life, made without any clear consciousness of their grounds, logical necessity is implicitly present as an immediate feeling of certainty. In cases of this kind we simply identify ourselves with the judgment, and *feel* that it is impossible that it can be false. But of course no judgment can claim to be necessary in its own right. Its necessity comes from its connection with other facts known to be true. Or in logical terms, we may say that it comes from reasons or premises supporting it. And one should always be ready to show the grounds or reasons upon which one's feeling of necessity rests. But in ordinary life, as we have said, it is not unusual to regard a conclusion as necessary without clearly realizing the nature of the reasons supporting it. An uneducated man is rarely able to go back and discover the reasons for his belief in some other statement of which he is convinced. If you question his assertion he feels that you are reflecting upon his veracity, and consequently grows angry. In the feeling of immediate necessity or conviction he identifies himself with the judgment and does not see that the criticism is not directed against the latter, but against the grounds by which it is supported.

In this distinction between necessity that is merely *felt* and the necessity that is *conscious* of its own grounds we see the direction in which judgment must develop. In the evolution of thought we gradually become conscious of the grounds upon which our judgments are made. That is, the simple judgment that seems to stand in isolation is seen to expand so as to include its reasons as an organic part of

itself. By itself it is only a fragment of a more complete and widely embracing thought. The feeling of necessity is an evidence of its dependence and connection, though this dependence and connection upon other facts may not be clearly understood. But what is implicit must be made explicit; the necessity that is merely *felt* to belong to the simple judgment must be justified by showing the grounds or reasons upon which it rests. And for this purpose the simple judgment has to be brought into relation with other facts and judgments outside of it, yet constitute its reasons, or are necessary to support it. In other words it must develop into an inference. As a matter of fact the same form of words as used by different persons, or by the same person at different times, may express either a judgment or an inference. Thus 'the price of wheat rose after the war began' might express either a simple historical fact accepted from experience or from hearsay, or it might, in the mouth of a person acquainted with the laws of supply and demand, be the necessary conclusion of a number of premises. Again, a child might read that 'the travelers found great difficulty in breathing when they reached the top of the mountain', accepting this as a simple statement of fact. If he were to read this same statement some years later, however, he would probably connect it at once with other facts regarding the nature of the atmosphere and the action of gravity, and so perceive at once its inferential necessity.

According to the view just stated necessity is not a property belonging to any judgment in itself, but something arising through its dependence upon other judgments.

In other words necessity is always mediate, not immediate. This view, however, differs from a theory that was once generally received, and has some adherents even at the present time. In dealing with the facts of experience we always explain one fact by referring it to a second, and that second by showing its dependence upon some third fact, and so on. Thus the movement of the piston-rod in an engine is explained by the pressure of steam, and this is due to the expansive power of heat, and heat is caused by combustion of fuel, etc. We are thus referred back in our explanations from one fact or principle to another, without ever reaching anything that does not require in its turn to be explained.

Now it is said that this process cannot go on forever; for if it did there could be no final or complete knowledge; the whole system would be left hanging in the air. There must, therefore, it is argued, be some ultimate facts which furnish the support for the world of our experience, some principle or principles which are themselves necessary and do not require any proof. That is, there must be certain propositions which are *immediately* necessary, and which serve as the final explanation for everything else. Now it is clear that such propositions must be entirely different in character from the ordinary facts of experience, since their necessity belongs to their own nature, and is not derived from any other source. It had to be supposed, therefore, that they stood upon a different plane, and were not derived from experience. To explain the superior kind of certainty which they were assumed to possess, it was supposed that

they were present in the mind at birth, or were innate. They have also been called *necessary truths*, *a priori truths*, and *fundamental first principles*, in order to emphasize their supposed distinction from facts which are derived from experience.

When one regards knowledge as an internal process of growth or development, however, where each element plays its part as do the members of a living body, the inadequacy of any view which looks for a mechanical basis for knowledge is apparent. What is present in experience is a moving system of functions, not a structure of fixed mechanical parts such as exist, for example, in a building.[1]

§ 87. **Judgment Involves Both Analysis and Synthesis.** — The business of our thought is to understand the ways in which the various parts of the real world are related. And a judgment, as we have already seen, is just a single act of thought — one step in the process of understanding the world. Now we ask: How does Judgment accomplish its task? Does it proceed althogether by analysis, by pointing out the parts of which things are composed, or does it also employ synthesis in order to show how various parts combine in such a way as to form a whole? Or is it possible for both of these processes to be united in one and the same act of judgment?

Suppose that one actually makes the judgment for oneself (and does not merely repeat the words of the proposition), 'the rose has pinnate leaves'. What has taken place? We notice, first, that a new property of the rose has been

[1] See above, pp. 192 ff., 244 ff.

brought to light; a distinction or mark has been discovered in the content 'rose' which was not seen to belong to it before the judgment was made. So far the process is one of analysis, of discovering the parts or distinctions of something which is at first taken as it were in a lump. And this is a most essential element in all thinking. In order to know it is absolutely necessary that the differences between the parts of things should be clearly apprehended, that we should not confuse things which are unlike, or fail to make proper distinctions. If we examine a number of instances where a real judgment is made we shall find that this moment of analysis or discrimination is always present. Sometimes indeed, analysis may not seem to be the main purpose of the judgment; but if one looks closely one will always find in a judgment that elements which are unlike are held apart or discriminated.

But let us look again at the same judgment, 'the rose has pinnate leaves'. It is not difficult to see that the discovery of something new in itself is only one part of what the judgment has accomplished. It also affirms the union of this new discovery with the properties of what we call the rose. It is therefore from this point of view an act of synthesis. It asserts that the prickly branches, fragrant flowers, feather-like leaves, and other distinctions are united in the one content which we call the rose. It does not stop with the mere assertion, 'there is a mark or distinction', but it affirms that it is a mark of something, *i.e.*, that it is united with other marks or properties to form a concrete whole. In other words we may say that every judgment affirms

the *unity* of the different parts or aspects of a thing; and this is of course synthesis. From this point of view Judgment can be defined as a process of synthesis, just as we defined it above as one of analysis.

But how, it may be asked, is it possible for a judgment to be both analytic and synthetic? Are not these processes directly opposed to each other? It is true that there can be no doubt that this is the case when we are dealing with material things: pulling things to pieces is the opposite of putting them together. When we are doing the one we cannot also be doing the other. But there is no such opposition between these processes when they go on in our minds. An illustration may make this point clear. Suppose that one is trying to understand some piece of mechanism, say a watch; in order to be able to see how it goes, or judge correctly regarding it, two things are necessary. First, one must notice all the parts of which it is composed — the wheels of various sizes, springs, pins, etc. But in the second place, one would not understand the watch until one saw how all the parts were united, how one part fits into another, and all combine together into one whole. We do not mean that these are two steps which take place in succession; as a matter of fact the detection of the various parts and the perception of their connection go hand in hand. In the process of understanding the watch we have both taken it to pieces and put it together again at one and the same time. In the world of material things, as we have said, only one of these processes could go on at a time; but in every act of thinking, in every judgment,

analysis and synthesis go hand in hand, and one has no meaning except with reference to the other.

But the two moments or factors of analysis and synthesis, although present in every judgment, are not always equally prominent. The main purpose of the judgment usually falls on one side or the other. In a judgment like 'water can be divided into hydrogen and oxygen' the main emphasis seems to be on the parts, and the assertion that these elements are *parts of a whole*, though present, is only implied. But when one asserts 'these springs and wheels together make up a watch' it is the nature of the whole upon which the emphasis is laid, and the separation or discrimination of the parts is, as it were, secondary. It is not difficult to see, however, that the two moments of Judgment are present in both cases. The difference consists in the fact that at one time analysis, and at the other synthesis, is made the main purpose.

It was at one time supposed that analytic and synthetic judgments were entirely different in kind from each other. This view is of course fundamentally different from the account of Judgment that we have just given. The absolute distinction between analytic and synthetic judgments, like the theory that thought begins with concepts, arises from a substitution of the spoken or written proposition for the judgment itself. In the proposition the subject seems to be the starting-point. We have a word or term which appears to be independent and capable of standing alone. The question is, then, where shall we find the predicate? For example in the proposition 'iron is an element' the subject

stands first and the predicate comes later. It seems possible then to say that we have first the subject, 'iron', and then join on to it the predicate, 'element', which has been obtained either by analyzing the subject or from some previous experience. But the proposition as a collection of words must not be substituted for the act of judgment. Judgment, as we have already seen, is a single act of intelligence which at once discriminates and brings into relation different aspects of the whole with which it is dealing. A mere subject by itself has no intelligible meaning. If one hears the word 'iron', for example, it may call up certain mental images; but by itself it is not a complete thought or fact in which we can rest. 'Well, what of it?' we say. The mind at once goes on to form some judgment like 'this is iron' or 'iron is heavy'. We cannot *think* a term without thinking something *of* it. In short, although the words forming the subject of a *proposition* are relatively independent and can be used without the words making up the predicate, in a *judgment*, on the other hand, a subject is only a subject *through* its relation to a predicate. The proposition may be divided into parts but the judgment is a single thought-activity, and cannot be divided (cf. § 84).

§ 88. **Judgment as Constructing a System of Knowledge.** — In this section we have not to take account of any new characteristic of Judgment, but rather to emphasize the part it plays in building up knowledge. As we have seen, Judgment works both analytically and synthetically: it discovers new parts and distinctions and at the same time brings the parts into relation and thus builds up a whole.

That is the law according to which thinking develops and is just what we called differentiation and integration in a previous section (§ 82).

It is necessary here, however, to dwell upon the fact that each judgment may be regarded as a step in the process of building up a *system* of knowledge. The emphatic word here is 'system', and we must be perfectly clear about its meaning. A system is a whole composed of various parts. But it is not the same thing as an aggregate or heap. In an aggregate or heap no essential relation exists between the units of which it is composed. In a heap of grain, or pile of stones, one may take away any part without the other parts being at all affected thereby. But in a system each part has a fixed and necessary relation to the whole and to all the other parts. For this reason we may say that a building, or a piece of mechanism, is a system. Each stone in the building, each wheel in the watch, plays a part, and is essential to the whole. In things which are the result of growth the essential relations in which the parts stand is even more clearly evident. The various parts of a plant or an animal have their own functions, but at the same time they are so necessary to one another that an injury to one is an injury to all. We express this relation in the case of living things by saying that the parts are *organic* to one another. And in the same way it is not unusual to speak of society as an organism, in order to express the fact that the various individuals of which it is composed are not independent units but stand in necessary relations to one another and are all mutually helpful or hurtful.

We have said that judgment constructs a *system* of knowledge. This implies that it is not merely a process of adding one fact to another, as we might add one stone to another to form a heap. Judgment combines the new facts with which it deals, with what is already known, in such a way as to give to each its own proper place in relation to and interdependence with the others. Different facts are not only brought together, but they are arranged, related, systematized. No fact is allowed to stand by itself but has to take its place as a member of a larger system of facts and receive its value and meaning from this connection. Of course a single judgment is not sufficient to bring a large number of facts into relation in this way. But each judgment contributes something to this end and brings some new fact into relation to what is already known. Even in a simple judgment like 'that was the twelve o'clock whistle' the constructive or systematizing work accomplished is evident. The auditory sensation in itself, as a mere sound, not a piece of knowledge at all, is interpreted in such a way as to find a place in the system of experience. One may appreciate what part the judgment really plays by remembering how the sound appeared before one was able to judge. There may have been at first a moment of bewilderment — 'What does this mean?' one asks. In the next moment the judgment is made: 'It is the twelve o'clock whistle'. That is, our thinking has constructed a meaning for it, and brought it into relation with the rest of our knowledge.

Every new experience is thus brought into relation with

the facts we already know, and is tested by them. It has to find its place in the system of knowledge — to join itself to what is already known. If this is impossible, if what claims to be a fact is entirely opposed to what we already know on the same subject, it is usually declared to be false. Thus we would refuse to believe that some person whom we know well and respect was guilty of theft; for it would be impossible to connect such conduct with what we already know of his character. And similarly we find it impossible to believe, even although we have the evidence of our senses, that the conjurer has actually performed what he professes; for to do so would often be to reverse entirely our conception of natural laws. It must not be forgotten, however, that the existing system of knowledge, which seems to serve as the standard and test of new facts, is itself undergoing constant modification through the influence of these facts. As new experiences are brought into connection with the existing body of our knowledge there is a constant rearrangement and readjustment of the latter going on. Usually this adjustment is slight and takes place almost imperceptibly. But in some cases a single fact may be so significant as completely to transform what seemed to be the accumulated knowledge of years. The experiment which Galileo made by dropping balls of different weight from the tower of Pisa made it impossible to hold any longer the old theory — which seemed as certain as anything well could be — that the velocity with which bodies fall is proportional to their weight. Again, if theft were actually proved against the man we respect, that single

fact might be sufficient to force us to give up everything we supposed we knew about his character.

We have said that judgment is the process by which knowledge grows into a system. It is by judging or thinking that we attempt to bring the various parts of our experience into relation with one another. The degree to which this has been done is the measure of our intellectual development. The knowledge of the uneducated and unthinking man, like that of the child, is largely composed of unrelated fragments. It is an aggregation, not a system of facts. The facts which go to make it up may quite well be contradictory, but this contradiction is not seen because no attempt is made to unite them. There is of course no human experience which is entirely systematic, or which has been completely unified. Even those who have thought most deeply find it impossible to fit together exactly knowledge gained from different fields and from different sciences. The facts of one science, for example, may seem to stand by themselves and not to have any relation to the facts derived from another science. Or there may appear to be a conflict between the results of physical sciences and the truths of moral philosophy and religion. But the ideal always remains, that truth is one and indivisible, and that it must be possible ultimately to harmonize all facts in one all-embracing system of judgments.

EXERCISES (XXI)

1. Why is the statement that 'there is no other standard of truth than the momentary judgment of the individual' the *reductio ad absurdum* of scepticism?

2. Truth is the standard, both of itself, and of falsehood. Explain this statement.

3. If there are no innate truths does it follow that all our knowledge must be derived from experience? Explain.

4. Show how the systematic character of knowledge accounts for the several characteristics of judgment described in this chapter.

CHAPTER XXII

THE LAWS OF THOUGHT

§ 89. **The Law of Identity.** — We found (§ 83) that judgment is the simplest form of thinking. And in the last chapter we were engaged in studying its main characteristics, and becoming acquainted with its mode of operation. The essential nature of the thinking process, therefore, has already been stated, though we have not traced the mode of its development or shown its application to the various problems of experience. But before undertaking this it is necessary to turn aside to consider another problem. In nearly all books dealing with logic one finds a statement of three fundamental laws of thought differing greatly, in form at least, from what we have so far learned regarding the nature of judgment. These laws are so well known by name, and yet so ambiguous in their mode of statement, that it seems well to try to decide what meaning to apply to them. For their interpretation will be found to furnish further illustration of the nature of judgment and will thus throw light on the discussions of the last chapter. The Laws of Thought are usually regarded as axioms, or propositions requiring no proof, rather than as laws descriptive of the nature of thought in any special circumstance. In this sense they are supposed to be the foundation of all logic, since they are presupposed in all thinking.

The first of these laws, or axiomatic principles, is that of **Identity.** 'Whatever is, is', 'Everything remains identical with itself'; 'A is A'. These are some of the forms in which the law is usually stated. What is meant by these statements is, that in all argument, we necessarily assume, if we are to reason at all, that each thing possesses a permanent character and does not pass now into this, now into that *at random.* If any knowledge is to be possible at all the character of things must remain fixed. Socrates is always to be Socrates, and iron, iron. Things are also constantly undergoing changes. The law of Identity, of course, does not deny this, or declare that the changes are unreal. It rather presupposes the changes; but goes on to affirm that there is an *identity persisting in and through the difference.* Identity means *identity in difference:* it is this which all our judgments assert. Socrates changes, or is different from day to day and from year to year. But he also remains identical with himself; he is in his old age the same Socrates who talked with Parmenides in his youth and fought at Potidæa when in middle life. Identity, then, does not affirm the static and unchangeable character of things and thoughts; but that there is *continuity* in change, in virtue of which things maintain themselves and are capable of being known as parts of a coherent system. Every one assumes as much as this in every judgment he makes, though he may not himself be conscious of it.

Another interpretation of this principle was, however, offered by Boole and Jevons, who developed what is known

as the Equational or Symbolic logic. According to these writers the law of Identity expresses the fundamental nature of judgment, and is to be interpreted as a statement of an exact and bare identity. That is to say, every categorical judgment is the expression of an identity between the subject and the predicate. The judgment 'New York is the largest city in America' is simply a case of *a* is *a*. It expresses the fact, that is, that New York and the largest city in America are identical. 'Iron is a metal' is another example of the same principle. It may be written: iron = metal. And since the copula may often be ambiguous it will be better to discard it in working out arguments, and adopt in its place the sign of equality.

Judgment, from this point of view, is thus simply an equation, and may be written as such. Furthermore, the conclusion of a series of logical premises may be obtained by a process similar to that employed in working algebraic equations. That is, we can substitute for any term in a judgment its equivalent, or the value which it has in another judgment. This method Jevons calls 'the substitution of similars', which he maintains is the fundamental principle of all reasoning.

If now we employ letters to symbolize the terms of the propositions, it is claimed that we can work out any argument by the equational method. Take the syllogism: —

All metals are elements,
Iron is a metal,
Therefore iron is an element.

Now represent metal by M, iron by I, and element by E. Then the argument in equational form will be,

$$M = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$I = M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and by the substitution in (1) of the value of M in (2) we get I = E, the required conclusion.

Or we may illustrate this method by a somewhat more complex example which is also taken from Jevons: 'Common salt is sodium chloride, which is a substance that crystallizes in cubical form; but what crystallizes in cubical form does not possess the power of double refraction'. The conclusion of this argument may be found by letting A = Common Salt, B = Sodium Chloride, C = something which crystallizes in cubical form, and D = something which possesses the power of double refraction. The negative of any of these terms will be expressed by the corresponding small letters. The argument may now be expressed: —

$$A = B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$B = C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$C = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By substitution of the value of C in (2) we get,

$$B = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

And substituting here the value of B in (1),

$$A = d.$$

Giving to these symbols their meanings, we get the result 'common salt does not possess the power of double refraction', as the conclusion of the argument.

Of course, in simple arguments like those we have been examining, there is nothing gained by the use of symbols and the representation of arguments in this form. But when the various terms employed are much longer and more complex, simplification may be attained in this way. Various other symbols have also been used to express the relation of the various terms to one another, and a symbolic logic has been developed which follows very closely the procedure of algebra. By following closely the methods of mathematics, but seeking to obtain a more general form of expressing the relations than mathematics employs, results have been obtained that are of much interest and which may prove valuable.[1]

We have to ask, however, whether an equation represents fairly the nature of judgment. Does a judgment express merely the identity of subject and predicate? And if so, what *kind* of identity is referred to? In mathematical reasoning the sign of equality expresses the identity of quantitative units. When one says $2 + 3 = 5$ the meaning is that the number of units on each side of the equation is identical. And similarly the assertion that a parallelogram = two triangles with the same base and of the same altitude as itself expresses the fact that in the two cases the number of units of area, square feet, square yards, etc., is the same. In mathematics the equation declares that the quantitative relations of its two sides are identical. It does not assert

[1] The clearest statement of the aims and methods of the Equational Logic may perhaps be obtained from Jevons, *The Principles of Science*, Introduction. Cf. also G. Boole, *An Investigation of the Laws of Thought*, London, 1854; and C. I. Lewis, *A Survey of Symbolic Logic*, Berkeley, Calif., 1918.

that the two things compared — the triangle and one-half the parallelogram, for example — have the same qualities, or are exactly the same in all respects. Now if we extend the use of the sign of equality it must take on a new meaning. It is clear that in a judgment like 'iron = metal' there is no reference at all to quantitative relations. We are not asserting that the number of units in the two terms is identical. What, then, does the sign of equality express in such a case?

The answer is not difficult, say those who hold this theory. The sign of equality in such cases expresses *absolute identity;* the entire and complete sameness of subject and predicate. The proposition 'mammals = vertebrates' asserts that mammals and vertebrates are one and the same thing. But that statement in its present form is not true: the class mammal does not completely correspond with the class vertebrate. To make it exact, reply those who uphold the equational form, one must qualify or limit the predicate and write the proposition 'mammals = some vertebrates'. But even so, we may urge, the form of the judgment is still defective. In the first place it does not correspond to the model $a = a$. For one side, 'mammal', is clearly marked off, while the other is indefinite and vague. And secondly, just because of its vagueness, it is not a satisfactory piece of knowledge. To obviate these objections one must go further and write, mammals = mammalian vertebrates. At last the judgment seems to correspond to the type, $a = a$. But a new difficulty arises. Has not the judgment lost all its original meaning and become a mere tautology? There

seems to be no escape from the following dilemma: *either* there is some difference between subject and predicate and the judgment is therefore not in the form $a = a$, *or* the judgment is tautologous and expresses nothing. The view of the equational logic that judgment affirms the entire identity of subject and predicate refutes itself. The form $a = a$ cannot be regarded as the type to which all judgments conform.

But there must nevertheless be some kind of identity between the parts of a judgment. In one sense we do seem to declare that the subject and predicate are identical when we say 'iron is a metal'. As we have seen, however, if these terms are merely identical and nothing more the judgment loses all meaning. We are forced to the conclusion that every judgment affirms both identity and difference, or that there is identity running through and underlying the diversity. But is not this a paradoxical statement? When we affirm identity does not this imply the absence of all difference? If a is a how can it at the same time be something different from itself?

And yet this is just what every judgment which has any meaning affirms. 'Iron is fusible'. 'This table is made of oak'. 'The sword is rusty with age'. In all these judgments there is an assertion of the unity of different properties or parts in one whole. A is B and yet does not cease to be A is rather the type of judgment than a is merely or abstractly a. It is worth noticing that this view of the matter corresponds with the account of judgment already given. We saw that judgment constructs a *system* of knowledge by

showing that various things, which seem at first unrelated, are yet connected by an underlying unity. Knowledge is always the synthesis or union of different parts or different properties in a common identity. And each judgment, as an element of knowledge, displays the same essential structure which belongs to knowledge as a whole. It involves, as was shown in § 87, both analysis and synthesis, and declares the oneness or identity of a number of properties or parts, without at the same time losing sight of their distinctness.

Let us now sum up our discussion of the law of Identity. When rightly understood, as we have seen, it does not affirm that *a* can only be bare *a*, that the subject and predicate are absolutely identical. As a law of thought it expresses the fact that judgment brings together differences, *i.e.*, different things and qualities, and shows that they are parts of one whole or unity. That is, judgment reveals the underlying unity or identity which is present in the midst of variety. This law also states another characteristic of judgment which we have already emphasized. This is what we have called the universality of judgment (§ 85). It is to judgments, and not to concepts or terms, as has sometimes been supposed, that the law of Identity properly applies. What it affirms in this connection is simply that judgment claims to be true, and hence is identical at all times and for all persons. It cannot be true for you and false for me that 'iron is a metal', and the judgment must at bottom mean the same for all men. Truth is not a matter of individual taste, but every judgment which is true has a permanent character or identity of meaning belonging to it.

§ 90. **The Law of Non-Contradiction.** — The law of **Non-Contradiction** is the second of the so-called laws of thought. It is usually stated as follows: it is impossible for the same thing both to be *a*, and not to be *a*, or, *a* is not *not-a*. It is evident that this law states in a negative form the same characteristics of thought as the law of identity. Indeed it was in this form that the principle was first laid down by Aristotle. "It is impossible", he says, "that the same predicate can both belong and not belong to the same subject at the same time, and in the same sense."[1] We cannot assert that Socrates is both wise and not wise. Truth is not, as the Sophists supposed, a matter of taste or convenience, but must be consistent with itself. If a judgment affirms that 'iron is a metal' it at the same time excludes the assertion that it is not a metal. There is a fixity and permanence about judgments which prevents them from changing into anything else. And it is just this permanence which we have already called the universality of judgment, which the law of Non-Contradiction expresses in a negative form.

This law has however sometimes been interpreted in such a way as to make it equivalent to the assertion of abstract or bare identity as described above. That is, the statement that it is impossible for any judgment to unite *a* and *not-a* may be taken to mean that it is impossible to assert the unity of *a and anything different from a*. But as we have seen, this is exactly what we do in every judgment

[1] *Metaphysics*, Bk. III, Ch. IV. See also the remaining chapters of the same book for Aristotle's demonstration that all thought presupposes such a principle.

which is more than a tautology. The law, then, does not forbid the union of *differences* in one judgment, but of *contradictories*, or of what would destroy the integrity of the judgment and render it unmeaning. If the law is to hold true of judgment, *not-a* must not be taken as equivalent to anything simply different from *a*, but as signifying what is opposed or contradictory to *a*.

It is not by any means easy to decide what things are merely different and therefore compatible with one another, and what contradictory or opposed. Logic can give no rule which may be applied in every case. If experience shows that two things or two properties are at any time united, we say that they are merely different from each other; if they have never been found in conjunction and we are not able to conceive how their union could take place, we call them opposites or contradictories. It is worth noticing, too, that no terms are *in themselves* contradictory except those which are in the form *a* and *not-a*, wise and not-wise. But they *become* contradictory and exclude each other when they claim to occupy the same place in some particular system of facts. Thus 'maple' and 'oak' denote trees of a different variety, which are, however, so little opposed that they may exist side by side. If both these terms were applied to the same tree they would of course become contradictory. By claiming to stand in the same relations these terms become rivals, as it were, and exclude each other. But a knowledge of the particular facts involved is always necessary in order to determine whether or not two assertions are really incompatible.

§ 91. **The Law of Excluded Middle.** — The third law is a corollary from what has just been said in the last section. There is no middle ground, it declares, between contradictories. A is *either b or not-b.* To affirm the one is to deny the other. When we have real contradictories — *i.e.*, when *not-b* is not merely something different from *b*, but something which excludes it — every judgment is double-edged, and both affirms and denies at the same time. To deny that the throw of a penny has given heads is to assert that it has fallen tails. As we have seen, however, logic affords no rules for deciding when things do thus stand in the relation of mutual exclusion. The law of Excluded Middle states only that *where* this relation does exist every proposition has a double value and both affirms and denies at the same time. It requires special knowledge of the particular facts in each case to enable us to decide what things are thus opposed to one another. There is no logical law by means of which things may be divided into two contradictory or exclusive groups or classes.

It is important to notice that all of the judgments which we use in everyday life are to some extent double-edged. That is, they contain, besides what is directly affirmed, some implication or counter statement. For example, to say 'that object is red' is implicitly to deny that it is blue, or any other color. The statement 'A never looks at a book' carries with it certain implications which may perhaps be held in mind as a series of hypotheses: 'Is he then too busy, or sick, or simply indifferent'? In almost any field where we have any systematic knowledge we can limit pretty

definitely the number of possibilities — *a* must be either *b*, or *c*, or *d*. In such cases to affirm that *a* is *b* is of course to deny implicitly *c* and *d;* and conversely, the denial of any one possibility, as *c*, enables one to assert that *a* is *b* or *d*. In ordinary conversation misunderstandings and misconceptions frequently arise because neither party is fully aware of all the possible cases and the relation between them. It is very difficult, however, to make a statement which will have no counter implications. If one says 'this railway system does not employ steam power' the proposition seems to justify the question: 'Does it then use electricity or compressed air'? We should feel that it was a mere quibble if the person who made the statement should reply: 'I did not say it employed any kind of power'. 'There are some small errors in this paper' would ordinarily be taken to imply the counter proposition, 'the paper contains no serious errors'. It is clear that it is only when one's knowledge becomes *systematic* — *i.e.*, when one knows the relations in which all the facts in the field under consideration stand to one another — that one can be fully aware of what is really implied in each assertion or denial (cf. § 88). It is however of fundamental importance to understand that in its work of defining the nature of things thought works with a double-edged tool. In other words, the process of elimination is not merely negative but yields positive results.

These so-called Laws of Thought, when read in relation to one another, may then be interpreted as expressing the universal Postulate of our intelligence, that experience shall

be capable of being organized as a system. If there were nothing but identity — if everything were identical with everything else — there could be no universe and no knowledge. Nor would any knowledge be possible if things were merely different: if there were no common space and time, no common natures and laws of relationship, the world would be nothing but a disorganized chaos, without form and void. Finally, experience would not be possible as a coherent system if each fact had not some particular place or bearing, in such a way that one affirmation or denial carried others with it. Reality exists as a system of mutual implications and exclusions. It must so exist if it is to be knowable. That Reality is knowable by Intelligence, may, then, be regarded as the ultimate postulate of knowledge, and this, as we have seen, is the final interpretation to be given of the Laws of Thought.

EXERCISES (XXII)

1. Explain the nature and function of the Laws of Thought.
2. Give original examples of their application.
3. Can these laws be proved? Defend your answer.
4. In what sense, if any, can a Law of Thought be violated? How does such a law compare with laws of nature and laws of the land in this respect?
5. Why is it sometimes held that the Laws of Thought supply only negative criteria of truth?
6. Show how the principle of Identity in Difference is related to the conception of knowledge as systematic.
7. What objections are there to employing symbols, such as the sign of equality, to represent the relation between the subject and predicate of a categorical judgment?

CHAPTER XXIII

TYPES OF JUDGMENT

§ 92. **Judgments of Quality.** — We have hitherto been considering the nature of judgment in general, and have learned something regarding its main characteristics. It is now necessary to examine briefly some of the more important forms or types of judgment. The different forms or conceptions in terms of which things are brought into relation are usually referred to as 'Categories'. This chapter might therefore have been entitled 'The Main Categories of Thought', as it is with certain typical ways in which things and their properties and attributes are related that we are here concerned. We shall begin with very simple and elementary ways of judging, and afterwards consider some of the more complex types. In this way we shall see the nature and structure of judgment illustrated at different levels of thought. We also hope to show, by this review of types, that there are no arbitrary divisions in the process of thinking, but that the lower forms of judgment gradually develop into the higher in accordance with the general law of evolution. It is of course impossible to carry out at present this plan in detail, for that would be to give a complete history of the development of thought. It will be necessary for us to take long steps and content ourselves with a general view of the relation of the various stages in the development of judgment.

The first efforts of intelligence to understand the world take the form of judgments of **Quality.** At a low stage of mental development it is the simple qualities of things which force themselves on attention. The young child, for example, takes notice only of the most obvious qualities of things. His judgments are very vague and indefinite. There is in them no discrimination of the various parts and relations of the objects, but they express merely a general impression based upon some striking quality. Thus it has often been noticed that the child calls every man 'papa', and any light, of whatever size, the moon. A little boy known to the author used to call Sisters of Charity crows, on account of the color of their dresses. The objects as he apprehended them were simply black, and nothing more. His intelligence rested in the qualitative total impression: the various parts, with their diverse relations, which he afterwards learned to know and distinguish, did not at that time exist for him.

It is perhaps impossible to find in the experience of an adult any judgments dealing entirely with simple qualities, and taking no account of the numbers, or even to some extent of the relations, of the parts. But we can find examples of judgment where the qualitative aspect is much the most prominent — where indeed the quantitative and more complex relations are scarcely noticed at all. 'This is green', 'that is a strange odor', 'there is something a long way off' — all these seem to be judgments of quality or general impression, and to involve scarcely any other element. This is also the easiest kind of judgment to make, the

judgment involving least mental effort, and noticing only the most evident, and, as may be seen, the most superficial, aspect of things. It is clear that such judgments belong to a lower stage of thinking than those implying analysis and perception of quantitative relations. Compare, for example, 'that is very large' with 'this tree is made up of roots, trunk, branches, and leaves'; or 'this is green' with 'this leaf is divided into two parts by a rib running through the centre'. The first judgment in each pair obviously involves much less intellectual work than the latter. The judgment of simple quality accordingly is, as we have said, the starting-point of thought. It is with this kind of thinking that the knowledge of the child begins. And before the savage learns to count, *i.e.*, to distinguish and enumerate the parts of the objects with which he deals, his judgments must necessarily belong to this same type.

It must never be forgotten, however, that simple judgments of quality are really judgments; that is, they are not given to the mind from any external source but are the products of its own activity. A judgment, as we have already pointed out (§ 83), implies a reaction on the part of the mind to what is presented to consciousness through the senses. It distinguishes and puts together the material which sense presents in such a way as to perceive its significance — what it really amounts to — as a piece of knowledge. This act of interpretative intelligence has gone, however, but a little way in the type of judgment with which we are dealing. But even in a vague qualitative judgment like 'there is something black' the essential characteristics

of judgment can be already distinguished. For it presupposes at least some analysis or discrimination of the black object from the rest of the environment, and of the black color from other colors. And the judgment 'something is black' has made at the same time a beginning in constructing this vague something into a system of qualities, or into a thing that is known. The other qualities and relations are as yet wrapped up in the indefiniteness of the 'something'. In spite of its indefiniteness, however, the latter plays the part of a permanent centre or identity. It is the whole from which the quality of blackness has been separated out, and to which it is again attached.

Our thought, however, is not satisfied with a knowledge of the general qualities of things, but pushes farther its work of analysis and construction. In this way it begins to distinguish the various parts of objects and to compare one with another. We not only judge that 'the grass is green', but go further and say 'this piece is dark green, and that light green'. The indefinite judgment 'this cane is heavy' is no longer satisfactory, and is replaced by 'this end of the cane is much heavier than that'. And when this stage is reached judgments of Quality are already passing into the next higher type, judgments of **Quantity.** For the element of comparison, already contained in these judgments, is the basis of counting, measuring, and all quantitative determination. In advancing from the simple apprehension of quality to the stage where it takes note of, and compares, the *degree* or *intensity* which the same quality manifests in different instances, intelligence has entered upon a path

leading directly to judgments of quantity. To distinguish parts, to regard things as degrees or instances of a common quality, is at once to suggest the quantitative process of counting and measurement.

§ 93. **Judgments of Quantity.** — It is very difficult, as we have seen, to draw a hard and fast line between quality and quantity. Indefinite judgments of general impression which do not imply any comparison seem always to be qualitative rather than quantitative in character. This is true, I think, of judgments like 'this object is very large', 'there was a great flock of sheep in the field'. In such cases the interest does not seem to be quantitative at all; *i.e.*, there is no effort made to determine *how many* units or parts there are in the whole about which the judgment is made. But the general impression of size or number is apprehended and judged of at the same level of intelligence, and in the same vague way, as the simple qualities with which we dealt in the last section. It is by means of such a general qualitative impression that the savage who cannot count beyond five is able to distinguish between six and some larger number. And we cannot imagine that the shepherd's dog learns that some of the sheep are missing by any process of counting. We must suppose that the general qualitative impression made by the smaller flock is different from that made by the larger, and that there has been no real counting or estimation of number in the case.

But quantitative judgments proper belong to a higher stage of intelligence than do those which have just been

described. Indefinite judgments, like 'this is very large', or 'there are a great many stars in that group', are not satisfactory pieces of knowledge. We accordingly set ourselves to get more exact information about the parts which compose the wholes, or to analyze and distinguish. The first step in this process leads to *Judgments of Enumeration.* If the whole which is analyzed is composed of homogeneous parts the judgments of enumeration take the form of simple counting. 'There are one, two, three, . . . twenty men in this company'. Where the parts are not of the same kind, however, a separate name may have to be given to each. 'This plant is composed of root, stalk, leaves, and flower'.

But exact quantitative knowledge requires us to do more than enumerate the parts of which a whole is composed. We must go on and *weigh* or *measure* them. There is of course no essential difference between weighing and measuring, so that we may call all judgments which express the result of this process *Judgments of Measure.* It is worth noting that judgments of this class are not so simple and direct as may appear at first sight. When we measure we express the relation of the parts with which we are dealing to some common unit or standard. The judgment 'this tower is 200 feet high' means that if the tower is compared with a foot-rule it will be found to be 200 times as long. It really involves a proportion and might be expressed:— tower: foot-rule = 200:1.

The point which it is important to notice is that all measurement is the result of comparison. In the first place some unit is more or less arbitrarily selected. Then the

judgment simply states the relation between this unit and the object measured: one is contained in the other once, or twice, or ten times. The quantitative determination thus obtained is accordingly merely relative. That is, it does not belong absolutely and in its own right to the object measured, *but indicates the relation of that object to something else.*

For this reason it may seem that quantitative relations tell us nothing regarding the real nature of objects, and that to discover what the latter are *in themselves* we shall have to return to the point of view of quality. But we have seen that simple judgments of quality yield a very vague and unsatisfactory kind of knowledge. Moreover, we should discover by thinking the matter out that even qualities always imply a reference to one another, and are no more absolute than quantities.

In order to obtain more satisfactory knowledge regarding things we shall have to go forward to a higher type of judgment, rather than backward to quality. But the importance of quantitative determination for exact knowledge must not be overlooked. By means of measurement things are reduced to common terms, as it were, and thus a basis of comparison is afforded where it would otherwise be impossible. To reduce everything to such a common measure is the business of the physico-mathematical sciences. Everything has a quantitive value and can be expressed mathematically in terms of some unit or standard, as, for example, the unit of heat, or of pressure, or the electrical unit. It was this tendency to count and measure and

weigh things which established the body of exact knowledge which we call science. And in almost every field knowledge increases greatly, both in extent and exactness, as soon as it is found possible to reduce the phenomena under investigation to a common measure, and to express their relations by means of mathematical formulas.

It is therefore a great step in advance to be able to compare things as quantities, and to express their relations in terms of number. But judgments of quantity are not entirely satisfactory; they are, as has already been noticed, merely *relative* in character. Moreover, from a quantitative point of view, each thing is equivalent to the sum of its parts. When the parts have been enumerated and measured the value of the whole is obtained by addition. But it is scarcely ever possible to represent adequately the nature of a whole in this way. So long as we are dealing with a piece of inorganic matter, the method of regarding the sum of the parts as equivalent to the thing, generally gives good results and leads to no difficulty. But it is quite different when the whole question belongs to something which has life and consciousness. In such cases we have what has already been called an organic whole (§ 88). Now it is clear that the principle of quantity, which can only add and subtract, is insufficient to represent completely the nature of an object of this kind. It has no means of representing the individuality or real whole, which rather constitutes the parts than is constituted by them. That is, to understand such objects, we shall have to take a new point of view, and begin with the whole rather than with the

parts. From the point of view of quantity the nature of the whole is discovered by adding together the parts; while in objects possessing an individuality of their own there seems to be a central principle to which the parts are subordinated, and in relation to which alone they can be understood. The type of judgments dealing with such objects we shall have to discuss in § 95.

§ 94. **Judgments of Causal Connection.** — Another class of judgments used in building up knowledge may be called judgments of **Causal Connection.** They undertake to show how the various changes which go on in things are connected causally with other things or events. This type of judgment — leading as it does beyond the particular object to a knowledge of the ways in which objects are connected — seems to belong to a higher stage of mental development than those which merely take note of quality and quantity. This does not mean that we never look for causes until the qualities and quantities of things have been discovered. Nor is it true that any causal judgment, however vague and unsatisfactory, is higher than any judgment of quality or quantity whatsoever. But in the beginnings of knowledge, one may say, thought does not travel outside the particular object to show the connections of the latter with anything else. And beginning in this way, it seizes first upon quality and quantity, which seem to belong to things in themselves. We have seen, however, that as a matter of fact judgments of quantity involve comparison, and so a reference of one thing to another, though that reference is not usually made consciously or explicitly. In this form of judgment the

reference does not seem to imply any objective relations of the things compared. If for example I say that this desk is twice as long as my arm, this relation appears quite external and accidental: the nature of the one remains independent of that of the other. But when we judge that one thing is causally connected with another the accidental relation expressed in quantity has become essential and objective, indicating a closer relationship between things than is expressed in a quantitative comparison of the judgment.

The word 'cause' has been used in a great many senses and its various meanings have given rise to a great deal of discussion. That every event must have a cause was formerly regarded as an innate truth, or *a priori* proposition. We have seen, however, that we do not come into the world with any ready-made stock of knowledge. All knowledge, we have often repeated, is the result of the mind's own judging activity. The so-called law of causation (every event must have a cause) must therefore express the fact that things are related as causes and effects. Intelligence is not satisfied to take things in isolation; it tries to gain an insight into the ways in which they are connected, to discover what one has to do with another. And this is just the characteristic of thought which was emphasized in § 88. Judgment, it was there said, is a process of constructing a *system*, of showing how the various parts of knowledge fit into one another, and are mutually dependent upon one another. The tendency of thought to connect things causally, then, is simply one of the fundamental forms in which its tendency

towards a system expresses itself. In employing the causal category judgment has become more explicit and conscious of itself than it was in quality and quantity.

It is interesting to note some of the more important changes which take place in the principle of causal explanation at different stages in the development of knowledge. The child and the savage regard all changes and events which take place in the natural world as due to the agency of living beings. These beings are represented as more or less similar to men, and as endowed with human passions and emotions. Thus we say that the earliest kind of explanation is essentially anthropomorphic. This word is derived from ἄνθρωπος, a man, and μορφή, shape or form, and hence is used to describe the way of representing either a spiritual being, as, for example, the Deity, or natural forces like fire, wind, etc., in human form. It is probably true that at a very early stage in the development of both the individual and the race every object is supposed to have life. Or perhaps it would be truer to say that the young child (and the same would be true for the savage on a low plane of intelligence) has not yet made the distinction between animate and inanimate objects, but vaguely regards everything as like himself. This first stage is usually known as *animism*, because each object is supposed to be endowed with a spirit, or *anima*.

Gradually, however, the distinction between animate and inanimate objects becomes clear. Accordingly we find that at a somewhat more advanced stage the mode of explanation takes a different form, though it is still anthro-

pomorphic. Physical objects are no longer regarded as having life in themselves; the changes in them are supposed to be due to the action of spirits separate from the objects, but who use them to accomplish their purposes. These invisible spiritual agents, to whom all natural events are referred, have been variously named. It is clear that the gods of mythology belong here, as well as the fairies, elves, ghosts, and witches of the popular folk stories. It was a great advance when a Greek thinker, named Thales, came to the conclusion that it does not in any way explain natural events to refer them to the action of the gods. For in the first place, to say that the gods cause this or that event, is to state something which we have no means of proving. And even if the assertion were true it would not really explain anything. For it would not enable us to understand *how* the changes in question came about. It would tell nothing whatever regarding the actual steps in the process itself. Thales saw this, and tried to give a *natural* explanation of the world and all that goes on in it. He tried to build up a real system of knowledge by attempting to show how everything which has happened in the world has been connected with some natural cause. We know very little about the actual explanation of the world which Thales gave, except that he tried to derive everything from water. It is on account of the method which he adopted, rather than of what he actually performed, that he is regarded as the founder of science. Thales first showed, one may say, that knowledge means an insight into the ways in which the actual phenomena of the world

are connected with one another. We cannot unite into a system things so different in kind as spirits and natural phenomena. Or we may say that real explanation demands that there shall be some likeness, or ground of similarity, between the cause and the effect. An event happening in the world of objects must be explained by showing its connection with some other event, of a similar character, on which it depends.

The development of this conception of scientific explanation also influenced still further the notion of causality. We have seen that in the beginnings of knowledge every event was supposed to be due to the action of some living agent or spiritual being. Even after this mythological mode of explanation is discarded and natural causes put in the place of spirits, it is still difficult to rid oneself entirely of the old anthropomorphism. The popular mind still tends to regard the cause as an *agent* which *produces* the effect, through some power or efficiency which it possesses. It is not necessary to raise the question at present whether there are any grounds for this belief. To discuss this problem would carry us beyond logic into metaphysics. What we wish to notice is that science has gradually abandoned the notion that the cause *does something* to the effect. That, as we have seen, is a remnant of the old pre-scientific idea, and a notion which does not aid at all in explaining phenomena. It is the business of science to show *how* the things and events which make up our experiences are necessarily connected with one another. Science has to discover what things invariably go along with one another and necessarily presuppose one

another. And when it is found that some particular thing or event, A, is invariably necessary for the appearance of another particular occurrence, B, the former is regarded as the cause and the latter as the effect. In order to eliminate as far as possible the notion of agency or efficiency which attaches to the word cause, the terms 'antecedent' and 'consequent' are often used to indicate this relation. For science the cause is not an active agent but the invariable and necessary antecedent of something else which simply follows it. The cause does not explain the effect by assigning an agent which brings the latter about through its personal efforts; but it explains, because it reveals another necessary step in the process, and gives us a new fact which joins on or can be connected with the one from which we start.

We conclude then that the cause of any event is its invariable and necessary antecedent. It has been already explained, however, (p. 280) that by antecedent is not meant merely what is prior to the effect in time. The word must be understood as signifying the essential condition or what is 'logically' prior. Temporal priority is often taken practically as an indication of logical priority, but the two relations cannot be identified. In another part of this book (Ch. XVI), it is shown what tests must be applied in order to determine whether two phenomena are merely accidentally conjoined, or whether the connection is essential and real. It is necessary now to take one more step in tracing the various ways in which the idea of causality has been used. As a result of a famous scientific discovery

made about the middle of the preceding century, a new element was added to the notion of cause in its application to physical phenomena. The law of the Conservation of Energy states that the amount of energy, or power of doing work possessed by any set of bodies, regarded as a closed mechanical system, remains constant. Any change in a material body is the result of a transformation of energy from one form to another. The same notion is applied to the world as a whole: it is assumed that the total amount of energy which it contains remains constant. All changes taking place in the physical universe — motion into heat, or electricity into motion — are regarded as simply different forms or manifestations of the one world-energy.

As a result of this law the effect always represents the same amount of energy, or power of doing work, as the cause. Since no energy is ever lost the one must be equal to the other. And as a matter of fact the quantitative equivalence of many of the various forms of energy has been proved by actual measurement. In working out this law, for example, Joule showed that "the energy stored up in the 1-lb. weight which had been pulled up 772 feet was gradually transformed, as soon as the weight was released, into an amount of heat capable of raising the temperature of a pound of water 1° Fahr.; while Hirn showed on the other hand, that exactly this amount of heat would, if it could be turned back again into energy, raise the 1-lb. weight to the height of 772 feet at which it stood before." [1]

The new element that this law adds to the idea of cause

[1] Buckley, *Short History of Natural Science*, p. 339.

as a necessary and invariable antecedent, is that of the *quantitative identity of cause and effect.* Taking the phenomena connected in this way to represent simply certain quantities of energy, we say that the one is equivalent to the other. The energy which the cause represents has been transformed without loss, and reappears in the effect. If what seems to be the total effect is not equal to the cause, part of the energy of the latter must have been transformed into something else as yet perhaps unnoticed. No energy can have been lost.

It therefore becomes the task of the physical sciences to show that this relation of quantitative identity exists between phenomena which are causally connected when these are regarded by the science as constituting a closed mechanical system. The ideal of physical science is to prove that two groups of phenomena are connected as cause and effect, by showing that both represent the same quantity of energy. For this purpose measurement and calculation are necessary. The physical sciences, as was pointed out in the last section, deal largely with judgments of quantity, and devote themselves to showing by measurement that the same amount of energy persists through the various changes which phenomena undergo. In establishing causal connections, therefore, the physical sciences find it necessary to use the principles of measurement and calculation.

It will be evident from what has been already stated that this relation of cause and effect should, in theory, apply to all phenomena whose energy is capable of being measured and represented in quantitative terms. As a

matter of fact, however, the law has been proved only in physics and chemistry. From the very nature of the case it is extremely difficult to measure exactly the relations of cause and effect in the sciences dealing with organic life. But even in those sciences the law of the Conservation of Energy is assumed to hold true. For example, the amount of energy which a plant contains is assumed to be exactly the same as that represented by the various elements or forces — water, sunlight, mineral substances, etc. — which were instrumental in composing it. In the same way we suppose that the same relation holds of the changes going on in the brain, though we are of course unable to prove this by actual measurement.

It is difficult, however, to see how this law can have any application to mental phenomena. We can indeed measure the intensity and duration of sensations. But neither feelings nor complex processes of mind seem to be capable of measurement in fixed and unambiguous units. Moreover, it is never possible to measure the energy, or power of doing work, which states of consciousness possess, and to equate one with another in this respect. And this being so, the law of the Conservation of Energy cannot, of course, apply to psychical causes and effects. In the mental sciences, then, we cannot claim that the notion of Causality contains the element of quantitative identity between cause and effect which has been found to exist in the physical sciences.

§ 95. **Judgments of Individuality.** — By **Judgments of Individuality** we mean judgments which regard some complex object as a real whole with a definite nature of its

own. Judgments of this kind are also frequently called judgments of **Purpose, or Teleology**. We have already had occasion (§ 87) to distinguish a mere aggregate or sum of parts, like a heap of stones, from a true whole possessing a certain character and individuality of its own. It is as aggregates rather than as true wholes that judgments of quantity and of causal connection regard objects. For these types of judgments are concerned with the parts — the former to measure them, and the latter to show their causal connection. It requires a new form of judgment to represent adequately the nature of a complex object possessing individuality. This form gives expression to the organic unity and wholeness of things, and emphasizes the way in which the parts coöperate for a common purpose or end. Thus we regard the parts of a plant as a unity coöperating in a common purpose, and a man as a conscious system of ends. The question as to whether it is allowable to employ any other category, or form of explanation, in science than that of causality, is of great importance. Certain biologists, at any rate, of whom J. S. Haldane may be taken as representative,[1] maintain that it is methodologically justifiable to assume as the fundamental biological reality

> not the separated parts of an organism and its environment, but the whole organism in its actual relation to environment, . . . defining the parts and activities in the whole in terms implying their existing relationships to the other parts and activities. We can do this in virtue of the fundamental fact, which is the foundation of biological science, that the struc-

[1] See his *Mechanism, Life and Personality* (New York, 1923), and other works.

> tural details, activities, and environment of organisms tend to be maintained. This maintenance is perfectly evident amid all the vicissitudes of a living organism and the constant apparent exchange of material between organism and environment. It is as if an organism always remembered its proper structure and activities; and in reproduction organic 'memory' . . . is transmitted from generation to generation in a manner for which facts hitherto observed in the inorganic world seem to present no analogy.[1]

How far, now, is it allowable to go in employing this teleological form of explanation in addition to explanation in causal terms? This question is too large to be discussed here, but it is suggested as of fundamental importance both for science and philosophy.

We have seen that judgments of causal connection relate phenomena as causes and effects. A change in an object is explained by showing that some other change or event invariably precedes it. But this change, in its turn, demands explanation, and has to be accounted for by the discovery of a new cause. This type of judgment shows that one phenomenon is connected with a second, a second with a third, and so on indefinitely. The view of the world which it presents is that of a never-ending series of causes and effects. It is never possible to find a cause which is not itself the effect of something else. No phenomenon possesses any independence of its own, but is simply a link in a series, or a piece of a whole that is never completed. We say,

[1] Quoted in Smart, *The Logic of Science*, pp. 158, 159. See the same work for further confirmation of this view.

therefore, that causal explanation leads to an infinite regress. The notion of a 'first cause' is then contradictory, if 'cause' be defined in the scientific sense, as a phenomenon existing in time and space.

In the last section it was stated that causal judgments connect one part of our knowledge with another, and in this way aid in uniting the parts of our experience in a systematic manner. Now it is undoubtedly true that it would be impossible to have any genuine knowledge of anything as a whole, or an individual, without knowing the way in which the parts are related and mutually depend upon each other. In that sense judgments of causal relation are indispensable to a knowledge of a true whole. Their relation to judgments of teleology or individuality is thus analogous to the relation of quantitative judgments to them. But this form of judgment taken by itself resolutely goes on connecting part with part — one phenomenon with another — and refuses to regard any group of parts as possessed of an independent character or individuality. From this point of view everything is externally determined; its cause, or principle of explanation, lies outside of it in something else. The mark of individuality, on the other hand, is the power of origination, or self-determination. If, then, there exist any genuine individuals, they are something more than causally determined phenomena. And it is suggested that in this sense of the word a biological organism, a man, social institutions such as a church or a university, the state, and so on, may be regarded as examples of individualities of greater or less degree of concreteness.

EXERCISES (XXIII)

1. Give original examples of each of the types of judgment described in this chapter.

2. 'Number is the first real thought'. Explain this statement.

3. Explain in your own words why a judgment of causal connection is of a higher type than a quantitative judgment.

4. What do we mean by an 'infinite regress', and what is its logical defect?

5. How do we seek to overcome this defect?

6. Discuss the question whether a judgment of individuality may take the form of a definition.

7. The task of philosophy has sometimes been defined as that of an evaluation and criticism of the Categories. Explain.

CHAPTER XXIV

THE NATURE OF INFERENCE — INDUCTION AND DEDUCTION

§ 96. **Judgment and Inference.** — It must not be forgotten that our object in these chapters is to obtain as definite a conception as possible regarding the nature of thought. To attain this end, we agreed (§ 81) that it would be advantageous to begin with the simplest or most elementary form of thinking. That form we found to be judgment. We have now endeavored to show what judgment is, and what part it plays in building up knowledge. And in the last chapter we have attempted to see some of the steps in the evolution of judgment, as it passes from simple judgments of Quality to judgments of Individuality. This account being completed, it remains now to discuss the nature of Reasoning, or **Inference,** as the process in which judgment occurs.

We shall probably get the clearest idea of the nature of Inference by regarding it as a completely developed judgment. As thinking develops from the form of simple judgment to that of Inference, it displays progressive differentiation and integration. In accordance with this law we can say (1) that inference is more complex than judgment. The latter process, in its simplest form, can scarcely be said to have any parts: it represents a single act or pulsation of intelligence. Inference, on the other hand, seems to imply

steps or stages in thinking — a passage of the mind from one fact to another. Moreover (2) inference differs from judgment in exhibiting the grounds upon which its statement rests. The simple judgment makes a declaration on the basis of sense-perception, as, for example, 'the mail-train has just gone down'; 'it rained yesterday'. Each of these statements stands alone, as it were; it does not attempt to gain support by pointing out the connection of the asserted fact with other facts. To infer, however, is just to show the necessary connection of facts — that from the presence or absence of certain things the presence or absence of certain other things necessarily follows. It is not necessary for inference that the conclusion reached should be a fact which was not hitherto known. We often do reach new truths by reasoning from necessary connections. Thus we might *infer* that the mail-train has just gone down, from the fact that this train is always on time, and that it is now five minutes past the hour. Or we might prove, to a person who doubted the correctness of our memory, that it rained yesterday, by pointing to other facts with which rain is necessarily connected. We might point to the muddy condition of the roads, the swollen streams, or, perhaps, might remind the person who questions the statement, that it was yesterday that A was out driving and came home soaking. In this way one tries to exhibit the *necessity* of the fact under consideration; and to do this is to infer.

But in the actual process of knowledge we more frequently go from a fact to its reasons than in the opposite direction. The intelligence begins by accepting all the connections as

true and universal which it meets with in ordinary experience, or which are suggested to it in any way. It does not trouble itself at all about the grounds of its judgments and thus the insufficient basis on which many of these stand is at first not evident. The child, for example, believes everything it is told by its mother or nurse, or, it may be, all the pleasant things it imagines. Very often, too, the judgments of older persons are determined by their own wishes. The man of sanguine temperament is quite sure that his project cannot fail to succeed. Another principle upon which both children and adults quite unconsciously proceed is that the future must always resemble the past. The child assumes that the order of events each day will be the same — that there will always be games after dinner, and visitors in the afternoon, because that has happened a number of times in the past. And one may have no better reason for believing that the sun will rise to-morrow than the fact that it rose yesterday and to-day.

In these early, unreflective judgments, the ground or principle upon which they are based is of course not conscious at all. Each judgment is accepted by itself and no questions are raised as to how it is known. But the development of intelligence may be regarded as a process of becoming conscious of the reasons which show the falsity of certain of our beliefs and the necessity of others. The original judgment is not in reality so isolated and unrelated as it appeared; it contains implicitly its own reasons. But the validity of its procedure cannot be made manifest until the reasons for the statement made by the judgment are brought to

light. In the development of knowledge the judgment must expand so as to show the reasons which it necessarily presupposes. In itself it is only a fragment of the complete statement, and it tries to complete itself by making clear the nature of the systematic whole which it involves, or to which it really belongs. It is not until the implicit reasons which every judgment contains are thus brought to consciousness that it can be either proved or disproved. Taking the mere judgment by itself it is only possible to place one man's assertion against another's denial. But proof or disproof of a proposition implies that reasons are given for or against it. If its connection with some fact, or set of facts, known to be true, becomes evident on reflection, the *felt* necessity that the judgment possesses (§ 85) is transformed into a logical necessity. But if no such connection can be found, or if the judgment in question is seen to presuppose propositions which are themselves false, we must cease to regard it as valid.

When a judgment develops so as to become conscious of its reasons, it has already taken on the form of inference. And as we have already seen, this is the usual procedure of knowledge. We begin by believing without reason, or we assume that certain things are true, and try to find reasons for our belief. The conclusion, which is of course logically last, is usually first for us, and we set out from it to find the grounds, or the premises.

This way of proceeding from conclusion to premises, or from a judgment to its reasons, implies however that the mind is already aware of the distinction between false knowl-

edge and true, and therefore that the work of criticising and testing knowledge has already begun. The criticism of knowledge is probably forced upon the mind at first by the practical consequences of false judgments. So long as false judgments lead to no unpleasant results they are likely to pass unnoticed, without any question being raised regarding the grounds by means of which they are supported. The child usually believes all that he is told until he discovers that his credulity is making him a laughing stock, or has led to the loss of some pleasure. Sooner or later he learns that the ground upon which he has been unconsciously proceeding — somebody told me — is insufficient. In the same way the natural tendency to regard all the connections we happen to find existing between events as universal and necessary becomes more critical and discriminating. The child soon learns that the events of one day do not necessarily follow in the order of the day before, and that it is not always rainy on Fridays and fine on Sundays. But in order to discriminate between what is true and what is false he is obliged to go beyond the facts themselves and to become more or less clearly aware of the grounds assumed in each type of judgment. He is forced to include in the judgment the reasons by which it is supported. And in this way the distinction between valid and invalid principles of connection is gradually learned. Through experience, more or less dearly bought, we learn that we cannot depend upon hearsay, and also that many of the most obvious connections between events are not essential and have no claim to be regarded as universal laws. It becomes evident that

in order to reach true principles of connection it is necessary to take a wider survey of the facts and to push the process of analysis further than is done by our ordinary judgments of sense-perception. For example, we may at one time have supposed it to be a universal law that hot water will break glasses when poured into them. But as soon as we have experience of any instance or instances to the contrary we see that there is no essential connection between hot water and broken glasses. We find that we must go behind the obvious facts of the case in order to discover what is the real antecedent in the two cases. The two instances — where the glasses break, and where they do not — *seem* to be the same; and yet, since the result is different, there must be a difference which further analysis will bring to light, such as the greater thickness of the glasses which break. It is by penetrating beneath the point of view of ordinary knowledge that science endeavors to show how phenomena are really and essentially connected.

§ 97. **The Nature of Inference.** — We have seen that it is difficult to draw any hard and fast line between judgment and inference. In general, however, we may be said to reason when we do not simply accept a fact on the basis of sense-perception or memory, but show that it necessarily follows from some other known fact or facts. Inference, then, requires (1) that certain data or premises should be accepted as already known; and (2) it implies an insight into the necessary connection of some new fact or set of facts with what we already know. Thus one is said to infer B when one sees that it necessarily follows from some

fact already known. It is not necessary for an inference that B should never have been in consciousness before. As we have seen in the last section, what we very often do in inference is to show the reasons or necessity of some fact that we have previously accepted without knowing why. No matter whether we go from premises to conclusion (from the reasons to the fact), or in the opposite direction, from the conclusion to the premises, we are said to infer whenever we find the ground for the existence of one fact in the nature of another fact. In the former case we use words like 'therefore' and 'consequently', to indicate the connection; or, when the reasons are stated last, we use 'for' and 'because'. Whenever these conjunctions are used correctly an inference has been made, and it is always useful in following a course of reasoning to make clear to ourselves precisely on what grounds it has been made.

Although inference seems very simple and very natural, its procedure is much more puzzling when looked at closely than one would at first imagine. As we have seen, there is no inference unless the result reached is different from the starting-point. But how are we ever justified in passing from a knowledge of one fact to another different from it? How can we ever pass from the known to the unknown? The Greeks, who loved to bring to light the paradoxes which so often underlie familiar facts, used to discuss this question. How is it possible for that which is unknown — external to the mind — to pass into the mind and get itself known? It was to solve this puzzle that Plato pro-

pounded the doctrine that all knowing is remembering. Knowledge, he declared, is not increased by learning that of which we were altogether ignorant, but by a process of calling to mind or recollecting the knowledge which the soul possessed in a previous state of existence, but which was forgotten when it entered upon the conditions of the present life. It was therefore not necessary to suppose, according to Plato, that the mind performed the impossible feat of knowing what is external to itself, or that things previously unknown pass bodily into our minds and thus become known.

Plato was undoubtedly right in protesting against the popular view that knowledge is received into the mind in mechanical fashion, as food is received into the stomach. Knowledge, as we have frequently seen, is built up from within, and not put in from without. But the apparent paradox of knowledge may be explained without adopting Plato's poetical notion of a previous state of existence. We may admit that the process of inference would be quite inexplicable if it proceeded from one fact, A, to a knowledge of a second fact, B, which is totally different from the former. When we examine cases of inference, however, we find that there is always a certain amount of identity between the two ends of the process. The conclusion is always different, and yet not entirely different from the premises. Thus from the propositions 'all metals are elementary substances' and 'gold is a metal', one can infer that gold is an elementary substance. It is possible to connect 'gold' and 'elementary'. Here the identical link — what is called in formal

logic the middle term — is 'metal'. It is possible to connect gold and elementary substance, because the former is at the same time a metal, which in its turn is an element. Of course these conceptions — gold, metal, element — are not absolutely identical; it was pointed out in § 89 that propositions cannot be regarded as expressing mere identity without difference. But we can say that there is a common thread or element running through these notions and furnishing the principle of connection. Where we cannot discover such a common nature no inference can be made. Thus for example it would be impossible to draw any conclusion from the statements that 'it rained yesterday' and 'gold has been discovered in Alaska', because there is no common element or connecting thread present which would lead us beyond the premises.

In formal arguments the middle term, or connecting link, is usually explicitly stated; but in the actual process of reasoning things out it is frequently necessary to go in search of it. We may notice, for example, that the fire in a stove burns more slowly when the damper is shut. In order to understand the fact we have to find out some fact which is common to 'closed-damper' and 'slow-burning', some link of identity, as it were, enabling us to pass from the one to the other. Such a connecting link is afforded in this case by the supply of oxygen. Darwin was noted for his keenness in detecting connections which escape the ordinary eye, as well as for his skill in giving explanations of them. On one occasion he observed that in the part of the country where he lived, clover was abundant in those fields which were

situated near villages, while the outlying fields were almost destitute of it. What now, he asked himself, is the connecting link between these facts? Some investigation of the matter convinced him that the three agencies which produced this result were humble-bees, mice, and cats. The bees fertilize the clover flowers and thus make the plant abundant, the field mice destroy the bees' nests, but the cats go out from the villages into the fields near by and kill the mice.

We have seen that the passage from one fact to another in inference does not involve a transition to something wholly different from the starting-point. There is always some aspect or feature in which the premises are identical with the conclusion. And it is on the strength of this identity that a passage can be made from one to the other. The same fact may be expressed differently by saying that all inference takes place within a system, where the parts are so held together by a common nature that you can judge from some of them what the nature of the others must be. Suppose you were given the leaf of a plant. If you had some systematic botanical knowledge it might be possible to infer the species of plant to which the leaf belonged. That is, from the nature of a part, the nature of the whole to which it belongs could be determined. The part represents the whole — in some sense contains it implicitly. It is said that the great naturalist Cuvier could determine by examining a single tooth the nature of the animal to which it belonged. Let us suppose that the tooth were that of a ruminant animal. Now a zoölogist who knows the charac-

teristics of such an animal could draw various inferences regarding the possessor of the tooth. He could conclude, for example, that the animal to which it once belonged must also have had cloven hoofs. A single piece or part, that is, would enable one who knows accurately the system or common nature to which all the parts belong, to judge what the other parts are like.

The examples just given have referred to the possibility of an inference from one part of an organism to another. But as we have already seen, the systematic connection which here exists between the parts is more or less completely present whenever it is possible to infer at all. Inference pushes further the work of constructing a system begun by judgment (§ 88). If each thing were known by itself, if the parts of our knowledge did not fall together into systems where each part to some extent determines the nature of the other parts, no inference would be possible. It is because the various pieces of our knowledge are never independent of one another but form an organic whole, like the members of a living organism, that certain facts follow, as we say, from certain other facts. Otherwise we could only guess, or infer vaguely on the expectation that the future will resemble the past. Even this expectation, however, has no rational basis unless the world does form some kind of a coherent system. It is of course true that practically a great deal of the knowledge of every one is unsystematic, being composed of facts and theories which have never been brought into relation. But knowledge is not to be described in terms of such defects in the case of individuals.

To understand it we must take it at its best and in its most complete form. It is obvious that as our knowledge in any field becomes more completely and exactly organized, it will be increasingly possible to use it as a basis for inference. The better we are able to put together in a systematic way the various facts which we have learned about geology, or astronomy, or the weather, the more *significant* each fact becomes. The geologist may be able to tell from the appearance of the cliffs what has taken place in a locality thousands of years ago. And similarly, for the fisherman, the temperature, direction of the wind, its rising or falling, etc., are all *signs* from which he is able to infer, more or less correctly, the kind of weather which may be expected. A person who had no systematic knowledge in either of these fields would, however, see nothing in the scarred rocks, or in the sudden changes of the wind; he might notice the facts, but would not be able to use them as a basis of inference.

It is important to notice that what has just been said goes to confirm our previous statements regarding the increasing degree of integration which knowledge shows in the course of its development. The knowledge of the scientist differs from that of the ordinary man, not only in the greater number of facts which the former contains, but also, as we have seen, in the degree of integration or coherence which these facts possess. Inference, then, is simply a deep insight based on definite knowledge into the necessary connection of things. It is an act of thought which discovers the essential relations between things which at first sight appear to have no connection with one another. As

has already been said, it is a reasoned judgment; *i.e.*, a judgment which has become conscious of the reasons for the connections which it affirms.

§ 98. **Induction and Deduction.** — It has been pointed out that there are two directions in which inference or reasoning may proceed. We may begin with certain facts or principles which are already known, or are assumed to be true, and proceed to show that some result necessarily follows from them. Thus we might infer from our knowledge of chemical principles that if the draughts of a stove are closed so that the supply of oxygen is lessened, the fire will burn slowly; or from the relative positions and revolutions of the planets, astronomical reasoning might lead to the conclusion that an eclipse of the sun will take place on a specified day and hour. This method of reasoning is known as deduction. It proceeds, as we have seen, from premises to conclusion. In the first part of this book this form of reasoning has been treated at some length and its rules of procedure stated. At present we need only notice that in deductive reasoning the particular case is always brought under some general law or principle already known or assumed as true. Socrates is known to be mortal because as a man he falls under the general law that all men are mortal; the closing of the draughts is a case of lessened supply of oxygen, and therefore, in accordance with the general law, a case of slow burning. A deductive inference shows what are the results of the application of a general law to particular facts or instances. It proceeds downwards, as it were, from the general law to its consequences.

In Induction, on the contrary, we begin with particular phenomena and try to discover from them the law or principle which unites them. Certain facts are observed to happen together, and the problem is to find the ground or explanation of this connection. Inductive inference is thus a process of reading the general law out of the particular facts, of transforming the hypothetical answer to the problem into a systematic principle or theory. It is an insight into the nature of the whole or system, based upon a careful examination of the parts. 'Yesterday the smoke tended to fall to the ground, and it rained in the afternoon'. These two facts may simply be observed a number of times without any thought of their connection. But intelligence asks: Why should they happen in conjunction? And to answer this question we must begin by analyzing the facts in our possession. When the smoke falls to the ground the atmosphere must be lighter than usual; this is the case when it contains a great deal of moisture; but when the atmosphere is in this condition it usually tends to discharge its moisture in the form of rain: therefore we have the general law which enables us to show that the behavior of the smoke and the rain yesterday were not only accidentally *conjoined*, but essentially *connected.*

Deduction and induction, then, are both forms of inference, but the starting-point and mode of procedure of the one is different from that of the other. Consequently it is not unusual to speak of them as two *kinds* of reasoning which are quite distinct and independent of each other. It is important to avoid this popular error, and to remem-

ber that the real process of inference is in each case the same. The essence of inference, as has been shown, consists in the fact that it exhibits the manner in which particular facts are connected together into a system or whole. And this end is achieved by both deduction and induction. In the former case the general law of connection — what we may call the nature of the system within which the particulars fall — is known, and we argue from this as to the nature and relations of the various parts falling within it. We have the common thread which unites the various facts in our hand, and following it out are able to show its application in determining the nature of events which have not yet come within the range of our experience. Knowing the law of gravity, for example, one could infer deductively what momentum a ball weighing one pound must necessarily have after falling one hundred feet. It would not be necessary actually to measure the momentum of the falling body in this particular case, but it could be shown to be the necessary result of the general law. What the deductive inference shows us is the way in which a general principle or law of connection runs through a group of facts and constitutes them a real or organic whole. The same insight is reached by inductive inference, although the starting-point is entirely different. As we have already seen, induction begins by observing that certain phenomena are frequently conjoined and attempts to discover some law or principle which will make the fact of their connection intelligible.

It is usual to say that in induction we go from the par-

ticular facts to the general law. The following, however, would be a more correct form of statement: Before the inference we observe that a number of phenomena occur together, but do not know whether this conjunction is necessary or not; or, if we assume that it is necessary, we do not understand why it should be so. As a result of the inductive inference we gain an insight into the necessary connection of the observed phenomena, and also understand the principle according to which the latter are united. What we really obtain through an inductive inference is not only a general law but also a perception of its concrete application to particular phenomena. This being so, it is clear that induction and deduction are not two different kinds of inference. Inference always implies an effort on the part of the mind to see how phenomena are necessarily connected according to some general principle. And in carrying out this purpose the mind must begin with the knowledge it already possesses. When the general law of connection is known and the object is to discover the nature of some particular fact, the method of procedure is deductive. But when the problem by which we are confronted is to read out of the facts of sense-perception the general law of their connection, the method of inference which must be employed is that of induction. But from whatever point we set out, and whatever may be the immediate object of the inference, the result is always the same — an insight into the necessary connection of facts according to some general principle. And both in ordinary life and in scientific procedure induction and deduction

are constantly employed together as mutually supplementing each other in the work of organizing experience.

§ 99. **Science and Philosophy: Conclusion.** — Throughout the preceding chapters thinking has been described as the function through which the organization of experience is achieved, or as a process of building up a system of knowledge. It has become clear that the development of thinking involves a continuous increase in both differentiation and integration, and that these two moments or aspects of thought are organically related to each other. An advance in knowledge implies at once new facts and distinctions and also the perception of new connections and relations among facts. The ideal of completed knowledge, accordingly, would be a system of truths in which the place and meaning of every fact would be completely defined, and where, at the same time, the complete relation of every fact and every group of facts to every other would be fully exhibited. Nothing would then be indefinite for knowledge, and nothing would be isolated; to know things in this completely systematic way would be to see the world steadily and to see it whole.

Like all ideals, this conception is never completely realized in experience as we know it. This however does not render it idle or without practical significance. In the first place, it has importance as indicating the direction in which the further development of knowledge must proceed. And secondly, it is only by reading our actual knowledge in the light of the end towards which it is progressing that

we are able to understand its nature. That is, as stated in the first section of this book, thinking has to be defined as the function, or system of functions, whose end and goal is *knowledge*. Now knowledge is only attained in so far as unification and system are attained: the essence of knowledge is not found in its lack of system and definiteness — these are its defects and privations — but the cognitive experience of any individual has a right to the title of knowledge just in so far as these conditions are realized.

The problem of how a more complete unity of knowledge than that realized in the results of the special sciences is to be attained thus becomes of the highest importance. We may use the term Science to denote the entire work of discovery and systematization of facts which is carried on by the various civilized nations through successive generations and centuries. In this inclusive sense Science is undoubtedly one of the greatest achievements of the human race, and one of the highest objects of endeavor for the individual. Within this one body of knowledge, however, it is possible to make various distinctions between different sciences and groups of sciences. The various sciences might be classified, for example, as more or less abstract, or as more or less inclusive in character. Or again, the sciences of nature might be distinguished from the humanistic or social sciences, dealing with the distinctive products of man's life and thought, as shown, for example, in religious, social, or political institutions, or in art, science, and philosophy. But the division of the complete body of knowledge (Science, *Wissenschaft*) with which we are here directly

concerned, is that between the sciences and philosophy. For Philosophy is the name given to the endeavor to reach some rational unification of the knowledge derived from the various forms of experience, and especially from the various sciences. "Knowledge of the lowest kind", said Herbert Spencer, "is un-unified knowledge; science is partially-unified knowledge; philosophy is completely-unified knowledge." We may accept this statement with the understanding that of course no knowledge is entirely un-unified, and that, on the other hand, no actually existing system of philosophy can claim to have achieved an entirely complete and satisfactory unification of knowledge.

Now the systematic interpretation of the nature of the real world has been divided into various fields of investigation. Each science takes as its subject-matter a definite field or group of phenomena and endeavors to describe and explain, as accurately as possible, the facts that fall within that field. Thus, for example, astronomy studies the heavenly bodies with the purpose of making clear and comprehensible their changing phases and relations; botany deals with the various forms and functions of plant life; history describes the significant events which have occurred during the past life of man in society. It is however not true that the sciences can be distinguished merely with reference to the nature of the particular field which they occupy. The same body of facts may be dealt with by a number of sciences; or rather, there are certain more general or fundamental sciences whose principles and results have to be

employed in the work of the more special fields of inquiry. In botany, for example, physical and chemical facts and laws are cited in order to render the behavior of the plant intelligible. In political economy, in like manner, one has to make constant use of history in the investigations which one undertakes. Nevertheless, even where two or more sciences seem to occupy the same field, it will be found that each has its own special way of reading the facts, so that strictly speaking the same phenomena are never studied in the same way or with the same purpose in view.

The question to be considered here, however, is the question of the relation of the special sciences to philosophy. It might appear at first sight as if the whole field of reality were occupied — or soon to be occupied — by the various sciences, and that no problem were therefore left for philosophy. But the very fact that each science is obliged, in order to render its investigations definite and fruitful, to limit the field of its inquiry, makes necessary some attempt to bring the results derived from the different fields into relation. And to correlate the results of these different scientific inquiries, which are gathered with various purposes, and often by the employment of quite different hypotheses, it is neither possible nor sufficient merely to set them side by side. The work that philosophy is called upon to undertake is to interpret these results in such a way as to render them coherent and mutually thinkable. Philosophy aims at unifying knowledge by finding a conception or set of conceptions which will enable us to think

the entire world as some kind of a consistent system. It seeks to satisfy our demand for a world-view, a *Weltanschauung*. When we take the widest and most accurate survey within our power of the facts of experience, what conclusions are we warranted in drawing regarding the whole system of things of which we are a part?

In attempting to find an answer to this most practical question it is of course necessary to take account of every well-authenticated form of experience and to give to each its proper place and value. This means, as we have just indicated, that we must seek to reconcile the findings of the several natural sciences with each other, whenever they appear to be contradictory. Even more important and difficult than this, however, is the task of reconciling, of synthesizing, the scientific view of the world with our social, æsthetic, and religious conceptions. A thorough analysis of the forms and methods of scientific inference, to which the present work might be regarded as an introduction, is manifestly a necessary means to the achievement of that synoptic view of the whole which Plato regarded as the final aim of philosophy. It is true that the majority of men never apply themselves directly to the solution of ultimate philosophical questions; but every one holds, more or less consciously, and in more or less definite form, some conception regarding the nature of the world and his own place in it. It is perhaps most frequently from theology or from literature that men derive their world-view, and they hold this, not as a reasoned system of knowledge, but rather through belief in authority, or on emotional

or æsthetic grounds. As distinguished from constructions of this character, philosophy aims at a reasoned system. Like the sciences it discards both emotion and tradition as guides, and proceeding by means of careful analysis and definition, it subjects all partial and one-sided views of the world to rational criticism. Its postulate is that there is nothing irrational, or from its very nature incomprehensible, in the nature of things. It is true that science and philosophy will never complete the work they are carrying on: the results arrived at are never final, but only starting-points for new investigations. But in the one case as in the other, the road is never barred; progress is always possible if the problem is formulated in an intelligible way. And when it is remembered that philosophy is the completion of the sciences, that the philosophical problem is the final problem of knowledge, the fact that neither the foundations nor its outlines are yet finally determined will not appear either strange or discouraging.

EXERCISES (XXIV)

1. Does the passage from judgment to inference illustrate the general law of logical evolution? Explain.
2. How is it possible to pass from the known to the unknown?
3. Explain under what circumstances only an inference is possible.
4. What is the common element in both induction and deduction? Would it be correct to describe induction as just the inverse of deduction? Explain.
5. Explain the distinction between 'Science' and 'the sciences'.
6. What part does philosophy play in the progress of knowledge towards unity?
7. Why would it be unsatisfactory to construct a philosophy simply

by taking as ultimate the most general laws and principles of physical science? Can you mention any philosophers who have proceeded in this way?

8. What is meant by the abstract or hypothetical character of the special sciences? Illustrate in the case of physics and psychology.

9. Explain the function of philosophy as the interpretation of the results of the sciences.

MISCELLANEOUS EXAMPLES OF DEDUCTIVE AND INDUCTIVE ARGUMENTS

MISCELLANEOUS EXAMPLES OF DEDUCTIVE ARGUMENTS

Arrange the following arguments whenever possible in regular logical order, supplying premise or conclusion where either is lacking, or condensing when several sentences are used to state one proposition; determine whether or not the arguments are valid; give the mood and figure of the valid categorical arguments; if any argument is invalid, point out and name the fallacy involved: —

1. Notes that produce beats are not harmonious. The fourth and fifth produce beats. Therefore they are not harmonious.

2. Every one desires happiness; virtue is happiness; therefore every one desires virtue.

3. God is beneficial. Good is also beneficial. It would seem, then, that where the essence of God is, there, too, is the essence of Good (Epictetus).

4. None but Democrats vote for Mr. B. All who vote for Mr. B. are Southerners. Therefore none but Democrats are Southerners.

5. Reduce to the first figure: — Falkland was a royalist and a patriot; therefore some royalists were patriots (Keynes).

6. The more correct the logic, the more certainly will the conclusion be wrong if the premises are false. Therefore where the premises are wholly uncertain the best logician is the least safe guide (Keynes).

7. The existence of sensations consists in being perceived;

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all objects are really collections of sensations; therefore their existence consists in being perceived.

8. Whoever believes this a heretic; so that you are no heretic, for you do not believe this.

9. If it be true, as Mr. Spencer thinks, that the past experience of the race has produced innate ideas and feelings, Weismann's denial of Use-inheritance would be refuted. Certainly, but it is just possible that Mr. Spencer's theory is not true.

10. In reply to the gentleman's arguments, I need only say that two years ago he advocated the very measure which he now opposes.

11. Haste makes waste, and waste makes want; therefore a man never loses by delay.

12. C is not D, for A is B; and I know that whenever A is not B, C is D.

13. What can be inferred from the premises: — Either A is B or C is D, Either C is not D or E is F? Exhibit the reasoning (*a*) in the form of a hypothetical syllogism, (*b*) in the form of a dilemma (Keynes).

14. What type of reasoning do the following arguments exemplify? Determine whether they are valid or invalid, and why.

(*a*) A is younger than B, B is younger than C, therefore A is younger than C.

(*b*) Spitzbergen is east of the North Pole, the North Pole is east of Alaska, therefore Spitzbergen is east of Alaska (Robinson).

15. If we cannot be sure of the independent existence of objects, we cannot be sure of the existence of other people's bodies, still less of their minds, since we have no grounds for believing in their minds except such as are derived from observing their

bodies. Thus if we cannot be sure of the independent existence of objects, we shall be left alone in a desert — it may be that the whole outer world is nothing but a dream, and that we alone exist (Russell).

16. Either the conclusion of an argument does contain something not given in the premises or it does not. Therefore inference is either useless or invalid.

17. Why does a ball, when dropped from the masthead of a ship in full sail, fall, not exactly at the foot of the mast, but nearer to the stern of the vessel? (Sellars).

18. What fallacy may be involved in calling a certain age 'the Reformation'?

19. If the imagination of man were the sole cause of his ideas, then it would be impossible that he should be able to apprehend anything, but he can apprehend something; therefore . . . (Spinoza).

20. If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease.

21. Show why IE is an impossible mood in all the figures of the syllogism, while EI is possible in all of them.

22. If acquired variations are transmitted, there must be some unknown principle of heredity; if they are not transmitted, there must be some unknown factor of evolution (Osborn).

23. The spectra of compound bodies become less complex with heat; but the spectra of the elements do not, since they are not the spectra of compound bodies.

24. What can you tell about a valid syllogism if you know: — (1) that only the middle term is distributed; (2) that only the middle and minor terms are distributed; (3) that all three terms are distributed?

25. If logic takes not merely the form but the matter of thought into account it must either consider all the objects of thought without distinction or make a selection of some alone. The former alternative is impossible, for if it were required that logic should comprise a full discussion of all cogitable objects, *i.e.*, if it must draw within its sphere all other sciences, and thus constitute itself in fact the one universal science, every one at once perceives the absurdity of the requisition and the impossibility of its fulfilment. But is the second alternative more reasonable? Can it be proposed to logic to take cognizance of certain objects of thought to the exclusion of others? As none but an arbitrary answer can be given to this interrogation the absurdity of this alternative is no less manifest than the other. The particular objects, or matter of thought, being thus excluded, the form of human thought alone remains as the object-matter of our science (Sir William Hamilton).

26. The figure of Tell cannot be historic, else he must have been mentioned by early historians, or his personality would be necessary to explain known facts of history.

27. No punishment should be allowed for the sake of the good that may come of it; for all punishment is an evil, and we are not justified in doing evil that good may come of it.

28. The express train alone does not stop at this station, and, as the last train did not stop, it must have been the express train.

29. Arrange the following so as to show the difference between the Aristotelian and the Goclenian sorites: —

The cost of labor depends upon the efficiency of labor; the rate of profits depends on the cost of labor; the investment of capital depends on the rate of profits; wages depend on the investment of capital; therefore wages depend on the efficiency of labor.

30. Animals only are sentient beings; all plants are insentient.

31. No reason, however, can be given why the general happiness is desirable, except that each person, so far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons (Mill, *Utilitarianism*).

32. Business enterprises are most successful when managed by those who have a direct interest in them; therefore enterprises carried on by the State are not likely to succeed.

33. All P is M; All S is M; therefore Some not-S is not-P.

34. If the orbit of a comet is diminished, either the comet passes through a resisting medium, or the law of gravitation is partially suspended. But the second alternative is inadmissible. Hence if the orbit of a comet is diminished, there is present a resisting medium.

35. How do we know that our intuitive beliefs concerning the world are invariably true? Either it must be from experience establishing the harmony, or an intuitive belief must certify the correctness. Now experience cannot warrant such harmony except in so far as it has been perceived. Still more futile is it to make one instinctive belief the cause of another. Thus we cannot know that any intuitive belief is universally valid (Bain).

36. Which of the following are real inferences? (1) 'This weighs that down, therefore it is heavier'; (2) 'This piece of marble is larger than that, and therefore is heavier.'

37. The parts of pure space are immovable, which follows from their inseparability, motion being nothing but change of

distance between any two things; but this cannot be between parts that are inseparable, which therefore must be at perpetual rest one amongst another.

38. All civilized peoples are progressive; all uncivilized peoples are superstitious; therefore some superstitious peoples are not progressive.

39. If a body moves, it must move either in the place where it is, or in the place where it is not. But a body cannot move in the place where it is, nor yet in the place where it is not. Hence a body cannot move at all.

40. Against what fallacy does the proverb, 'All that glitters is not gold', warn us?

41. When Crœsus was about to make war upon Cyrus, King of Persia, he consulted the oracle at Delphi, and received for an answer that, if he should wage war against the Persians, he would overthrow a mighty empire.

42. Your arguments against the philosophy of Hegel are of no value; for you uphold that of Schopenhauer, which is equally repugnant to common sense.

43. Our ideas reach no farther than our experience. We have no experience of divine attributes and operations. I need not conclude my syllogism. You can draw the inference yourself (Hume).

44. In how many ways can No S is P be proved syllogistically? Prove your answer from the general rules of the syllogism.

45. To-morrow afternoon, at four o'clock, the Rev. X. will deliver the third and last address of a series of plain talks to young men about their perils at the —— branch of the Y. M. C. A.

46. I have the right to publish my opinions concerning the present administration. What is right for me to do, I ought to do.

Therefore I ought to publish my opinions concerning the present administration.

47. You must be a friend to my friend because you are a friend to me.

48. Given A is B, to prove B is A. Now either B is A or not A. If B is not A, then we have the syllogism, A is B, B is not A, therefore A is not A, which is absurd.

49. If the plate had been originally on the outside of the ship I should have judged that there must be green paint on it; but I couldn't find green paint on that part of the ship.

50. It is possible to have in thought the conception of the most Perfect Being. This conception implies the reality of such a Being; for if the most Perfect Being as thus conceived has no real existence, then it would be possible to conceive of a still more Perfect Being which should possess reality, and thus the former would not be the most Perfect Being possible (St. Anselm's ontological argument).

51. All that we know or conceive are our own ideas. When, therefore, you say all ideas are occasioned by impressions in the brain, do you conceive this brain or no? If you do, then you talk of ideas imprinted in an idea causing that same idea, which is absurd. If you do not conceive it, you talk unintelligibly, instead of forming a reasonable hypothesis (Berkeley, *Hylas and Philonous*).

52. The principles of justice are variable; the appointments of nature are invariable; therefore the principles of justice are no appointment of nature (Aristotle).

53. According to Mr. Ross, the statement in the text (p. 168) that in a disjunctive argument the alternatives must be mutually exclusive leads to the following paradoxical result: Namely 'A is either B or C' and 'A is either not B or not C'

are identical in their import; for in each case the real alternants are 'B but not C' and 'C but not B'. Thus the two following propositions are on this view identical in meaning: 'Anyone who affirms he has seen his own ghost is either not sane or not telling the truth', 'anyone who affirms that he has seen his own ghost is either sane or truthful' (Keynes, p. 280, n.). What is the flaw in Mr. Ross's argument and illustration?

MISCELLANEOUS EXAMPLES OF INDUCTIVE ARGUMENTS

Analyze the examples of inductive reasoning given below, and point out what methods are employed, indicating also whether or not the conclusion is completely established, and naming the fallacy, if any be present: —

1. Some comets have been observed to have the same orbit as certain meteoric showers. The hypothesis is suggested that all meteoric showers may represent the dêbris of disintegrated comets. Biela's comet having been missing for some time, it was accordingly predicted that when next due it would be replaced by a meteoric shower. This prediction was verified by observation.

2. We have this, that, and the other hypothesis urged by different students as the cause of the great excavations of the Great Lake basins; and all of these are urged with force and with fact, urged justly to a certain degree. It is practically demonstrable that these basins were river-valleys antecedent to the glacial incursion, and that they owe their origin in part to the pre-existence of those valleys and to the blocking up of their outlets. And so this view of their origins is urged with a certain truthfulness. So, again, it is demonstrable that they were occupied by great lobes of ice, which excavated them to a marked degree, and therefore the theory of glacial formation finds support in fact. I think it is further demonstrable that the earth's crust beneath these basins was flexed downward, and that they owe a part of their origin to crust deformation. But to my judgment neither the one nor the other, nor the third, constitutes

an adequate explanation of the phenomena. All these must be taken together, and possibly they must be supplemented by other agencies (Chamberlin).

3. We should be guided by the decisions of our ancestors; for old age is wiser than youth.

4. Before it was known that light traveled in waves, it was known that sound did so. Light and sound were both capable of being reflected, and the direction of their reflection obeyed the same law, that the angle of reflection is equal to the angle of incidence. From these facts it was inferred that light, like sound, traveled in waves.

5. When a coin and a feather are dropped simultaneously in the receiver of an air-pump, the air being left in, the feather flutters to the bottom after the coin; but, when the air is pumped out of the receiver, the coin and the feather, being dropped at the same instant, reach the bottom of the receiver together.

6. It is a very suggestive fact that most of the stars belonging to the Milky Way have spectra of the solar type, which indicates that they are of the same general constitution as our sun, and are also at about the same stage of evolution; and this may well have arisen from their origin in a great nebulous mass situated at or near the centre of the galactic plane, and probably revolving round their common centre of gravity.

7. If the moon had been destined to be merely a lamp to our earth, there was no occasion to variegate its surface with lofty mountains and extinct volcanoes, and cover it with large patches of matter that reflect different quantities of light and give its surface the appearance of continents and seas. It would have been a better lamp had it been a smooth piece of lime or of chalk. It is therefore prepared for inhabitants, and similarly all other satellites are also inhabited (Sir David Brewster).

8. A chemist, as Mill observes, analyzes a substance, and assuming the accuracy of his results, we at once infer a general law of nature from 'a single instance'. But if any one from the beginning of the world has seen that crows are black, and a single credible witness says that he has seen a gray crow, we abandon at once a conjunction which seemed to rest upon invariable and superabundant evidence. Why is a 'single instance' sufficient in one case, and any number of instances insufficient in the other? (Mill).

9. Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyl of carbon-dioxide gas under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions of the leaves of an illuminated plant were covered with black paper, no starch was found in these portions.

10. When a tree, or a bundle of wheat or barley straw, is burnt, a certain amount of mineral matter remains in the ashes — extremely small in comparison with the bulk of the tree or of the straw, but absolutely essential to its growth. In a soil lacking, or exhausted of, the necessary mineral constituents, the tree cannot live, the crop cannot grow. Now contagia are living things, which demand certain elements of life just as inexorably as trees, or wheat, or barley; and it is not difficult to see that a crop of a given parasite may so far use up a constituent existing in small quantities in the body, but essential to the growth of the parasite, so as to render the body unfit for the production of a second crop. The soil is exhausted, and, until the lost constituent is restored, the body is protected from any further attack of the same disorder. Such an explanation of non-

recurrent diseases naturally presents itself to a thorough believer in the germ theory. . . . To exhaust a soil, however, a parasite less vigorous and destructive than the really virulent one may suffice; and if, after having by means of a feebler organism exhausted the soil, without fatal result, the most highly virulent parasite be introduced into the system, it will prove powerless. This, in the language of the germ theory, is the whole secret of vaccination (Tyndall). Have you any remarks to make on this explanation?

11. Fraunhofer in 1815, by means of a slit and a telescope, made the surprising discovery that the solar spectrum is crossed, not by seven, but by thousands of obscure transverse streaks. Of these he counted some 600, and carefully mapped 324. The same system of examination applied to the rest of the heavenly bodies showed the mild effulgence of the moon and the planets to be deficient in precisely the same rays as sunlight; while in the stars it disclosed the differences in likeness which are always an earnest of increased knowledge.

One solar line especially — that marked in his map with the letter D — proved common to several of the stars examined; and it was remarkable that it exactly coincided in position with the conspicuous yellow beam which he had already found to accompany most kinds of combustion. Moreover, both the *dark* solar and the *bright* terrestrial 'D-lines' were displayed by his refined appliances as double. In this striking correspondence was contained the very essence of solar chemistry; but its true significance did not become apparent until long afterwards.

12. Convincing evidence as to the true nature of the solar lines was however at length, in the autumn of 1859, brought forward at Heidelberg. Kirchhoff's experiment in the matter

was a very simple one. He threw bright sunshine across a space occupied by vapor of sodium, and perceived with astonishment that the dark Fraunhofer line D, instead of being effaced by flame giving a *luminous* ray of the same refrangibility, was deepened and thickened by the superposition. He tried the same experiment, substituting for sunbeams light from a Drummond lamp, and with similar result. A dark furrow, corresponding in every respect to the solar D-line, was instantly seen to interrupt the otherwise unbroken radiance of its spectrum. The inference was irresistible, that the effect thus produced artificially was brought about naturally in the same way, and that sodium formed an ingredient in the glowing atmosphere of the sun.

This first discovery was quickly followed up by the identification of numerous bright rays in the spectra of other metallic bodies with others of the hitherto mysterious Fraunhofer lines. Kirchhoff was thus led to the conclusion that (besides sodium) iron, magnesium, calcium, and chromium are certainly solar constituents, and that copper, zinc, and nickel are also present, though in smaller quantities.

These memorable results were founded upon a general principle first enunciated by Kirchhoff, which may be expressed as follows: Substances of every kind are opaque to the precise rays which they emit at the same temperature; that is to say, they stop the kinds of light or heat which they are then actually in a condition to radiate.

13. Baron Zach received a letter from Pons, a successful finder of comets, complaining that for a certain period he had found no comets, though he had sought diligently. Zach, a man of much sly humor, told him that no spots had been seen on the sun for about the same time — which was true — and assured

him that when the spots came back, the comets would come with them. Some time after that he got a letter from Pons, who informed him, with great satisfaction, that he was quite right, that very large spots had appeared on the sun, and that he had found a fine comet shortly after (De Morgan, *Budget of Paradoxes*).

14. It is an illusion in psychology and a corruption of logic to take the conditions which occasion the logical operations of thought for the operations themselves. There is only one delusion more desperate still, — to imagine that a complete physical theory of the nervous system will explain that which is itself the condition of any theory being possible at all (Lotze). Discuss.

15. "Science for the past is a description; for the future a belief; it does not show the necessity of any sequence of phenomena." Discuss.

16. Koch found that, while guinea-pigs, mice, and other animals were killed by inoculation with anthrax, birds were not affected. This invulnerability had very much struck Pasteur and his two assistants. What was it in the body of a fowl that enabled it thus to resist inoculations of which the most infinitesimal quantity sufficed to kill an ox? They proved by a series of experiments that the microbe of splenic fever does not develop when subjected to a temperature of 44° Centigrade. Now, the temperature of birds being between 41 and 42°, may it not be, said Pasteur, that the fowls are protected from the disease because their blood is too warm? Might not the vital resistance encountered in the living fowl suffice to bridge over the small gap between 41–42°, and 44–45°? . . . This idea conducted Pasteur and his assistants to new researches. 'If the blood of a fowl were cooled', they asked, 'could not the

splenic fever parasite live in this blood'? The experiment was made. A hen was taken, and after inoculating it with splenic fever blood, it was placed with its feet in water at 25°. The temperature of the blood of the hen went down to 37° or 38°. At the end of twenty-four hours the hen was dead, and all its blood was filled with splenic fever bacteria. But if it was possible to render a fowl assailable by splenic fever simply by lowering its temperature, is it not also possible to restore to health a fowl so inoculated by warming it up again? A hen was inoculated, subjected, like the first, to the cold-water treatment, and when it became evident that the fever was at its height it was taken out of the water, wrapped carefully in cotton wool, and placed in an oven at a temperature of 35°. Little by little its strength returned; it shook itself, settled itself again, and in a few hours was fully restored to health. The microbe had disappeared. Hens killed after being thus saved, no longer showed the slightest trace of splenic organisms. There have been great discussions in Germany and France upon a mode of treatment in typhoid fever, which consists in cooling the body of the patient by frequently repeated baths. The possible good effects of this treatment may be understood when viewed in conjunction with the foregoing experiment on fowls. In typhoid fever the cold arrests the fermentation, which may be regarded as at once the expression and the cause of the disease, just as, by an inverse process, the heat of the body arrests the development of the splenic fever microbe in the hen (Vallery-Radot, *Louis Pasteur*).

17. For many generations the people of the Isle of St. Kilda believed that the arrival of a ship in the harbor inflicted on the islanders epidemic colds in the head, and many ingenious reasons were devised why the ship should cause colds. At last it occurred to somebody that the ship might not be the cause of the

cold, but that both might be effects of some other common cause, and it was then remembered that a ship could only enter the harbor when there was a strong northeast wind blowing.

18. An eminent judge was in the habit of jocosely propounding after dinner, a theory that the cause of the prevalence of Jacobinism was the practice of bearing three names. He quoted, on one side, Charles James Fox, Richard Brinsley Sheridan, John Horne Tooke, John Philpot Curran, Samuel Taylor Coleridge, Theobald Wolfe Tone. On the other hand there were William Pitt, John Scott, William Windham, Samuel Horsley, Henry Dundas, Edmund Burke. Moreover, the practice of giving children three names has been a growing practice, and Jacobinism has also been growing. The practice of giving children three names is more common in America than in England. In England, we still have a King and a House of Lords; but the Americans are Republicans. Burke and Theobald Wolfe Tone are both Irishmen; therefore the being an Irishman is not the cause of Jacobinism. Horsley and Horne Tooke are both clergymen; therefore the being a clergyman is not the cause of Jacobinism. Fox and Windham were both educated at Oxford; therefore the being educated at Oxford is not the cause of Jacobinism. Pitt and Horne Tooke were both educated at Cambridge; therefore the being educated at Cambridge is not the cause of Jacobinism. The cause is, therefore, the having three names (Macaulay).

19. Newton showed that the bodies known as comets obey the law of gravitation; but it was by no means certain that the individual of the species observed by him in 1680 formed a permanent member of the solar system. With another comet, however, which appeared in 1682, the case was different. Edmund Halley calculated the elements of its orbit on Newton's

principles, and found them to resemble so closely those arrived at for comets observed by Peter Apian in 1531, and by Kepler in 1607, as almost to compel the inference that all three apparitions were of a single body. This implied its revolution in a period of about seventy-six years, and Halley accordingly fixed its return for 1758–1759. It punctually reappeared on Christmas Day, 1758, and effected its perihelion passage on the 12th of March following, thus proving beyond dispute that some at least of these erratic bodies are domesticated within our system, and strictly conform to its fundamental laws (Clerke).

20. Joule's experiments show that when heat is produced by the consumption of work, a definite quantity of work is required to produce that amount of heat which is known to the physicists as the unit of heat; the heat, that is to say, which is necessary to raise one gramme of water through one degree centigrade. The quantity of work necessary for this is, according to Joule's best experiments, equal to the work which a gramme would perform in falling through a height of 425 metres.

In order to show how closely concordant are his numbers, I will adduce the results of a few series of experiments which he obtained after introducing the latest improvements in his methods.

(*a*) A series of experiments in which water was heated by friction in a brass vessel. In the interior of this vessel a vertical axis provided with sixteen paddles was rotated, the eddies thus produced being broken by a series of projecting barriers, in which parts were cut out large enough for the paddles to pass through. The value of the equivalent was 424.9 metres.

(*b*) Two similar experiments, in which mercury in an iron vessel was substituted for water in a brass one, gave 425 and 426.3 metres respectively.

(*c*) Two series of experiments, in which a conical ring rubbed against another, both surrounded by mercury, gave 426.7 and 425.6 metres respectively.

Exactly the same relations between heat and work were also found in the reverse process; that is, when work was produced by heat (Helmholtz).

21. Loeb has shown that some animals, exposed to a ray of light, turn either towards or away from the source of light; and he has applied to such behavior the term 'heliotropism'; one long used by the botanists to denote the bending of plants towards the light. Hence, without more ado, he speaks of the 'establishment of the identity of the reaction of animals and plants to light', and reasons as follows: 'We have seen that, in the case of animals which possess nerves, the movements of orientation towards light are governed by exactly the same external conditions, and depend in the same way upon the external form of the body, as in the case of plants which possess no nerves. These heliotropic phenomena consequently cannot depend upon *specific* qualities of the central nervous system'. That is to say — having extended to certain reactions of animals the name 'tropism', which had been used to denote certain plant reactions to which they bear a purely external and superficial resemblance, Loeb holds himself justified in regarding reactions of these two classes as essentially similar or identical, although it is well known to him, as to everybody else, that they differ profoundly, if only in that a complex nervous system plays an essential part in the animal reactions, but is absent from the plants (McDougall).

22. Certain laws tended to make the conception of a material heat, or *caloric*, communicated by an actual flow and emission, familiar to men's minds. . . . But some steps have recently

been made in thermotics, which appear to be likely to overturn this belief, and to make the doctrine of emission as untenable with regard to heat, as it had before been found to be with regard to light. I speak of the discovery of the polarization of heat. It being ascertained that rays of heat are polarized in the same manner as rays of light, we cannot retain the doctrine that heat radiates by the emanation of material particles, without supposing those particles of caloric to have poles; an hypothesis which probably no one would embrace; for, besides that its ill fortune in the case of light must deter speculators from it, the intimate connection of heat and light would hardly allow us to suppose polarization in the two cases to be produced by two different kinds of machinery (Whewell).

23. It is the more necessary to be cautious in our attempt to identify the laws of light and heat, inasmuch as along with all the resemblances of the two agents, there are very important differences. The power of transmitting light, the *diaphaneity* of bodies, is very distinct from their power of transmitting heat, which has been called *diathermancy* by M. Melloni. Thus both a plate of alum and a plate of rock-salt transmit nearly the whole heat, the second stops very little of it; and a plate of opaque quartz, nearly impenetrable by light, allows a large portion of the heat to pass. By passing the rays through various media, the heat may be, as it were sifted from the light which accompanies it (Whewell).

24. In describing the result of a prismatic analysis of the voltaic arc formed between charcoal poles, M. Foucault writes: "Its spectrum is marked . . . in its whole extent by a multitude of irregularly grouped luminous lines: but among these may be remarked a double line situated at the boundary of the yellow and orange. As this double line recalled by its form and situa-

tion the line D of the solar spectrum, I wished to try if it corresponded to it; and in default of instruments for measuring the angles, I had recourse to a particular process. I caused an image of the sun, formed by a converging lens, to fall on the arc itself, which allowed me to observe at the same time the electric and the solar spectrum superposed; I convinced myself in this way that the double dark line of the solar spectrum coincides exactly with the double bright line of the arc. This process of investigation furnished me matter for some unexpected observations. It proved to me in the first instance the extreme transparency of the arc, which occasioned only a faint shadow in the solar light. It showed me that this arc, placed in the path of a beam of solar light, absorbs the rays D, so that the above mentioned line D of the solar light is considerably strengthened when the two spectra are exactly superposed. When, on the contrary, they jut out one beyond the other, the line D appears darker than usual in the solar light, and stands out bright in the electric spectrum, which allows one easily to judge of their perfect coincidence. Thus the arc presents us with a medium which emits the rays D on its own account, and which at the same time absorbs them when they come from another quarter. To make the experiment in a manner still more decisive, I projected on the arc the reflected image of one of the charcoal points, which, like all solid bodies in ignition, gives no lines; and under these circumstances the line D appeared to me as in the solar spectrum" (Dampier Whetham).

25. In the summer of 1840, Mayer, practising medicine in Java, was struck with the brighter red color of the venous blood of his patients. Reasoning on this he conceived it possible that the brighter color was due to less bodily oxidation being necessary to keep up the body temperature in hot climates.

This drew his attention to animal heat, thence to heat production in relation to mechanical work, and, finally, to all forms of force. From extensive researches along these lines he formulated the theory that throughout the universe, both in the inorganic and the organic world, there are forces which are convertible but are not destructible.

26. There are frogs which change their color from time to time. In looking for the cause of this phenomenon, the first step is to establish the occasion on which the change occurs, and the inquirer naturally restricts himself to such suggestions as might occur to a biologist. A number of these being disposed of by direct observation, according to the tests of presence and absence of the phenomena suggested to be connected, we reach the only reasonable suggestion left, namely, that which connects the change of color with the color of the surroundings. Further hypotheses as to the *modus operandi* of this connection are put forward, and one of these, otherwise not unreasonable, being excluded by the same test, it remains that the color-change involves the stimulation of the eye by light. Here two alternative interpretations of this condition are further suggested, awareness of color and a reflex mechanism. Awareness of color is excluded by observed color-change in a blind frog, and the suggestion of nervous excitation through a reflex mechanism as a circumstance common to the blind frog and the normal frog when changing color is accepted in so far as the cause and noted to be in harmony with other biological facts (Bosanquet, from Joseph).

27. A buttercup leaf, a blade of grass, a fern, a moss, a volvox, and a protococcus, all contain green coloring matter. I infer that all the members of the vegetable kingdom contain green coloring matter.

28. M. Arago, having suspended a magnetic needle by a silk thread, and set it in vibration, observed that it came much sooner to a state of rest when suspended over a plate of copper, than when no such plate was beneath it. Now, in both cases there were two true causes . . . why it should come to rest, *viz.*, the resistance of the air, which opposes, and at length destroys, all motions performed in it; and the want of perfect mobility in the silk thread. But the effect of these causes being exactly known by the observation made in the absence of the copper, and being thus allowed for and subducted, a residual phenomenon appeared, in the fact that a retarding influence was exerted by the copper itself; and this fact, once ascertained, speedily led to the knowledge of an entirely new and unexpected class of relations (Mill, *System of Logic*).

29. After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of thunder and lightning were the same as those produced by electricity. Lightning travels in a zigzag line, and so does an electric spark; electricity sets things on fire, so does lightning; electricity melts metals, so does lightning. Animals can be killed by both, and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another?

30. A leading expert in pathology remarks that "a chemist may, and frequently does, accept certain biological evidence as proved which we (pathologists) should reject as inconclusive, owing to the omission of certain controls and checks." What generalization regarding scientific methods and evidence does

this remark suggest, and how would you go to work to verify the generalization?

31. Construct an hypothesis to explain some fact of your experience, and explain how it may be either verified or overthrown.

32. In order to investigate the ability of insects to find their mates, Loeb arranged the following experiment. A female butterfly was placed in a closed and otherwise empty cigar box, which was then suspended from the ceiling of a room. The windows were opened. At the time, no other butterflies of this species were visible in the neighborhood. During the course of a few hours, however, several males of this species entered the room and alighted on the box. Would you feel justified in drawing inferences from this result?

33. Against what error in the formation of hypotheses was Laplace contending when, to Napoleon's observation that there was no mention of God in his work on Celestial Mechanics, Laplace replied that he had no need of that hypothesis?

34. Lord Curzon, arguing for the continued existence of a hereditary Chamber: "The hereditary principle is established in every branch and aspect of our national life. We have hereditary bankers, lawyers, and even hereditary cotton-spinners. Why should it be a blot and offence when applied to the House of Lords?"

35. The following is the cardinal passage in Harvey's famous argument for the circulation of the blood: "Let us assume either arbitrarily or from experiment, the quantity of blood which the left ventricle of the heart will contain when distended, to be, say, two ounces, three ounces, or one ounce and a half — in the dead body I have found it to hold upwards of two ounces. . . . Let us suppose as approaching the truth that the fourth, or

fifth, or sixth, or even that the eighth part of its charge is thrown into the artery at each contraction; this would give either half an ounce, or three drachms, or one drachm of blood as propelled by the heart at each pulse into the aorta; which quantity, by reason of the valves at the root of the vessel, can by no means return into the ventricle. Now, in the course of half an hour, the heart will have made more than one thousand beats, in some as many as two, three, and even four thousand. Multiplying the number of drachms propelled by the number of pulses, we shall have either one thousand half ounces, or one thousand times three drachms, or a like proportional quantity of blood, according to the amount which we assume as propelled with each stroke of the heart, sent from this organ into the artery; a larger quantity in every case than is contained in the whole body! . . . [Thus], supposing even the smallest quantity of blood to be passed through the heart and the lungs with each pulsation, a vastly greater amount would still be thrown into the arteries . . . than could by any possibility be supplied by the food consumed. It could be furnished in no other way than by making a circuit and returning" (*De motu cordis*, Ch. IX).

36. When Newton produced a bright spot on the wall of his chamber, by admitting the sun's light through a small hole in his window-shutter, and making it pass through a prism, he expected the image to be round; which, of course, it would have been, if the colors had been produced by an equal dispersion in all directions; but to his surprise he saw the image, or spectrum, five times as long as broad. He found that no consideration of the different thickness of the glass, the possible unevenness of its surface, or the different angles of rays proceeding from the two sides of the sun, could be the cause of this shape. He found also, that the rays did not go from the prism to the image in

curves; he was then convinced that the different colors were refracted separately, and at different angles; and he confirmed this opinion by transmitting and refracting the rays of each color separately (Whewell).

37. Borelli, with all his zeal for the exact mathematical treatment of physiological problems, assumed, being led to do so by reasons of analogy without attempting to make any direct observations on the matter, that a muscle during contraction was inflated, that it suffered increase in bulk. . . . Glisson confronted [this idea] with a single experiment, the result of which deprived [it] of all solid basis. He says: . . . Take an oblong glass tube of suitable capacity and shape. Fit into the top of its side near its mouth another small tube like a funnel. Let a strong muscular man insert into the mouth of the larger tube the whole of his bared arm, and secure the mouth of the tube all round to the humerus with bandages so that no water can escape from the tube. Then pour water through the funnel until the whole of the larger tube is completely filled, and some water rises up into the funnel. This being done, now tell the man alternately to contract powerfully and to relax the muscles of his arm. It will be seen that when the muscles are contracted the water in the tube of the funnel sinks, rising again when relaxation takes place. From which it is clear that muscles are not inflated or swollen at the time that they are contracting but on the contrary are lessened, shrunk, and subsided.

38. Some thirty years ago, a student of the Germanic languages, reading over an old English poem of considerable length, called the *Genesis*, was struck by the fact that five or six hundred lines, in the heart of the poem, seemed to differ in various respects from the lines which preceded and followed. Pursuing his inquiry further, and comparing the forms of these lines

with those of a kindred language, he came to the conclusion that this section, which had always been supposed to be original Old English, had in fact been translated from Old Saxon, and was therefore led to believe in the existence of an Old Saxon poem on this subject of Genesis, though he was obliged to confess that he found no other trace of its existence. Some twenty years after, another scholar, at work in the Vatican Library, which had only recently rendered its treasures more accessible, discovered a fragment of the missing Old Saxon *Genesis*, of which probably no one had read a line for a thousand years. Yet such had been the faith of competent scholars in Sievers's processes that no one was surprised when the missing manuscript swam into sight, any more than astronomers were amazed when the telescope pointed to the quarter of the heavens indicated by Adams and Leverrier, and revealed the planet Neptune, which no human eye till then had ever seen (Albert S. Cook, *The Higher Study of English*).

39. What inductive fallacy may David be said to have committed when he said in his haste that all men are liars?

40. If cathode rays are negatively electrified particles, then when they enter an enclosure they ought to carry into it a charge of negative electricity. This has been proved to be the case by Perrin, who placed in front of a plane cathode two coaxial metallic cylinders which were insulated from each other: the outer of these cylinders was connected with the earth, the inner with a gold-leaf electroscope. These cylinders were closed except for two small holes, one in each cylinder, placed so that the cathode rays could pass through them into the inside of the inner cylinder. Perrin found that when the rays passed into the inner cylinder the electroscope received a charge of negative electricity, while no charge went to the electroscope when the

rays were deflected by a magnet so as no longer to pass through the hole (J. J. Thomson).

41. This experiment proves that something charged with negative electricity is shot off from the cathode, travelling at right angles to it, and that this something is deflected by a magnet; it is open, however, to the objection that it does not prove that the cause of the electrification in the electroscope has anything to do with the cathode rays. Now the supporters of the ætherial theory do not deny that electrified particles are shot off from the cathode; they deny, however, that these charged particles have any more to do with the cathode rays than a rifle-ball has with the flash when a rifle is fired. I have therefore repeated Perrin's experiment in a form which is not open to this objection. . . . Two coaxial cylinders with slits in them are placed in a bulb connected with the discharge tube; the cathode rays from the cathode A pass into the bulb through a slit in a metal plug fitted into the neck of the tube; this plug is connected with the anode and is put to earth. The cathode rays thus do not fall upon the cylinders unless they are deflected by a magnet. The outer cylinder is connected with the earth, the inner with the electrometer. When the cathode rays (whose path was tracted by the phosphorescence on the glass) did not fall on the slit, the electrical charge sent to the electrometer when the induction coil producing the rays was set in action was small and irregular; when however the rays were bent by a magnet so as to fall on the slit there was a large charge of negative electricity sent to the electrometer. I was surprised at the magnitude of the charge; on some occasions enough negative electricity went through the narrow slit into the inner cylinder in one second to alter the potential of a capacity of 1.5 microfarads by 20 volts. If the rays were so much bent by the

magnet that they overshot the slits in the cylinder, the charge passing into the cylinder fell again to a very small fraction of its value when the aim was true. Thus this experiment shows that however we twist and deflect the cathode rays by magnetic forces, the negative electrification follows the same path as the rays, and that this negative electrification is indissolubly connected with the cathode rays (J. J. Thomson).

42. When the rays are turned by the magnet so as to pass through the slit into the inner cylinder, the deflection of the electrometer connected with this cylinder increases up to a certain value, and then remains stationary although the rays continue to pour into the cylinder. This is due to the fact that the gas in the bulb becomes a conductor of electricity when the cathode rays pass though it, and thus, though the inner cylinder is perfectly insulated when the rays are not passing, yet as soon as the rays pass through the bulb the air between the inner cylinder and the outer one becomes a conductor, and the electricity escapes from the inner cylinder to the earth. Thus the charge within the inner cylinder does not go on continually increasing; the cylinder settles down into a state of equilibrium in which the rate at which it gains negative electricity from the rays is equal to the rate at which it loses it by conduction through the air. If the inner cylinder has initially a positive charge it rapidly loses that charge and acquires a negative one; while if the initial charge is a negative one, the cylinder will leak if the initial negative potential is numerically greater than the equilibrium value (J. J. Thomson).

43. Sir Joseph Lister, the founder of aseptic surgery, states the origin of his method as follows: "When it had been shown by the researches of Pasteur that the septic property of the atmosphere depended, not on oxygen or any gaseous constituent,

but on minute organisms suspended in it, which owed their energy to their vitality, it occurred to me that decomposition in the injured part might be avoided without excluding the air, by applying as a dressing some material capable of destroying the life of the floating particles." At first he used carbolic acid for this purpose. The wards of which he had charge in the Glasgow Infirmary were especially affected by gangrene, but in a short time became the healthiest in the world; while other wards separated only by a passageway retained this infection.

44. To establish the fundamental law regarding the pitch of sound, Mersenne stretched a hempen rope over ninety feet in length, so that the eye could easily follow its displacements. It did not then emit any sound, but one could easily count the vibrations it made in any given time. He then shortened the cord by one half, and found it then made twice the number of vibrations in the same length of time. In reducing it to a third or a fourth of the original length, he observed that the oscillations became three and four times as rapid. He also made similar experiments, with like results, with a brass wire. He thus established the law that, all other things being equal, the number of vibrations of a cord is inversely as its length (Zahm, *Sound and Music*).

45. If I am not justified in general in inferring that *d* is a good book because *a*, *b*, and *c* are good books, why may I nevertheless conclude with some probability that *Guy Mannering* is a good book because *Waverley*, *Ivanhoe*, and *Rob Roy* are? What bearing has this on the question of induction by simple enumeration and the assertion that all inference is by means of a universal?

46. Slips of flexible and tough Muntz's yellow metal instantly become rigid and brittle when dipped into a solution of perni-

trate of mercury. Discuss the method by which this generalization might be reached from a single instance, and explain why in many other cases a large number of instances fail to yield a universal conclusion.

47. For there are only two possible *a priori* explanations of adaptations for the naturalist; namely, the transmission of functional adaptations and natural selection; but as the first of these can be excluded, only the second remains (Weismann).

48. The planet Mars resembles the Earth in possessing atmosphere, water, and moderate temperature, and we may therefore suppose it to be inhabited (St. Andrews).

49. "No Body can be healthful without *Exercise*, neither Naturall Body, nor Politique: And certainly, to a Kingdome or Estate, a Just and Honourable Warre is the true *Exercise*. A Civill Warre, indeed, is like the Heat of a feaver; but a Forraine Warre, is like the Heat of *Exercise*, and serveth to keepe the Body in Health" (Bacon, *Essays*).

50. Explain the procedure of the *reductio ad absurdum* form of argument.

51. After Becquerel's discovery of the photographic and electric activity of uranium, it was found that, like Röntgen rays, the rays from uranium produced electric conductivity in air and other gases through which they passed. In the year 1900, M. and Madame Curie made a systematic search for similar properties in a great number of chemical elements and compounds, and in many natural minerals. They found that several minerals containing uranium were more active than that metal itself. Pitch-blende, for instance, a substance consisting chiefly of an oxide of uranium, but containing also traces of many other metals, was especially active. When obtained from Cornwall its activity was about equal to that of the same weight of

uranium, but samples from the Austrian mines were found to be three or four times as effective. The presence of some more active constituent was thus suggested. To examine this point, the various components of pitch-blende were separated chemically from each other and their radio-activities determined. In this way three different substances, radium, polonium, and actinium, all previously unknown, appear to have been isolated by various observers (Whetham).

52. Since the days of Cavendish, the composition of the air had been looked upon as an ascertained fact; a certain proportion had been shown to be oxygen, varying amounts of carbonic acid and aqueous vapor were known to be present, while the remainder, as the result of careful investigation, was supposed to be nitrogen. Cavendish himself knew, so accurate was his work, that any undetected residue could not exceed the 120th part. But in the course of a long series of experiments, undertaken afresh to determine the densities of the principal gases, Lord Rayleigh detected a slight difference in the density of nitrogen as prepared from ammonia and as extracted from the air. This difference, amounting at first to about 0.1 per cent, was increased on subsequent more careful examination to nearly a half per cent. It was clear that the gases prepared by these two methods were not identical, and that some hitherto unknown body was responsible for the complication. The existence of this new body, the inert gas now known as argon, was announced by Rayleigh and Ramsay in 1894, and shortly afterwards it was isolated from its companion (Whetham).

53. In 1620 Jean Tarde argued that because the sun is "The eye of the world," and the eye of the world cannot suffer from ophthalmia, sun-spots must be due not to actual specks or stains on the bright solar disk, but to the transits of a number of

small planets across it. To this new group of heavenly bodies he gave the name of "Borbonia Sidera."

54. Galileo describes his invention of the telescope as follows: This then was my reasoning; this instrument [of which he had heard a rumor] must either consist of one glass, or of more than one; it cannot be of one alone, because its figure must be either concave or convex, or comprised within two parallel superficies, but neither of these shapes alter in the least the objects seen, although increasing or diminishing them; for it is true that the concave glass diminishes, and that the convex glass increases them; but both show them very indistinctly, and hence one glass is not sufficient to produce the effect. Passing on to two glasses, and knowing that the glass of parallel superficies has no effect at all, I concluded that the desired result could not possibly follow by adding this one to the other two. I therefore restricted my experiments to combinations of the other two glasses; and I saw how this brought me to the result I desired.

55. In Sir Humphry Davy's experiments upon the decomposition of water by galvanism, it was found that, besides the two components of water, oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. The insight of Davy conjectured that there might be some hidden cause of this portion of the effect: the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. . . . By the substitution of gold vessels for glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the

cause, whatever it was, was still in operation. . . . He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere decomposed by contact with the electrical apparatus. An experiment determined this: the machine was put under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and the alkali.

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